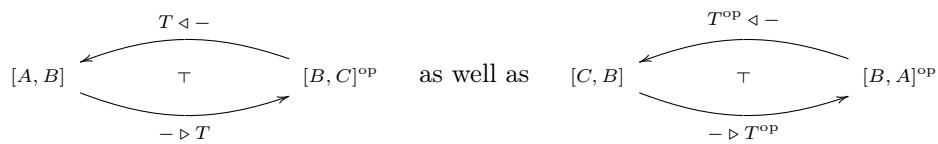


Algebraic Automata Theory

Sheet 1, 2017-11-09

Exercise 1 [10 POINTS]

Analyse the following polarities in \mathbf{rel} when $B = 1$.

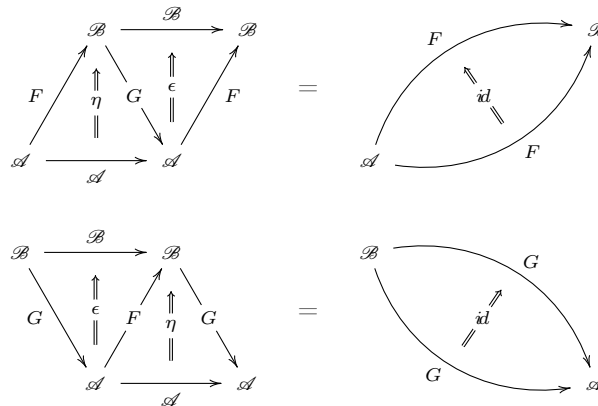


Turn them into adjunctions between the power-sets $A^{\mathbb{P}}$ and $(C^{\mathbb{P}})^{\text{op}}$. What happens, if the relation $A \xrightarrow{T} C$ is left-adjoint, *i.e.*, a function $A \xrightarrow{h} C$?

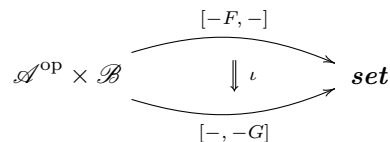
Exercise 2 [12 POINTS]

Show in detail that the two characterizations of adjunctions $F \dashv G$ in \mathbf{Cat} are equivalent:

(a) There exist natural transformations $\mathcal{A} \xrightarrow{\eta} FG$ and $GF \xrightarrow{\epsilon} \mathcal{B}$ subject to



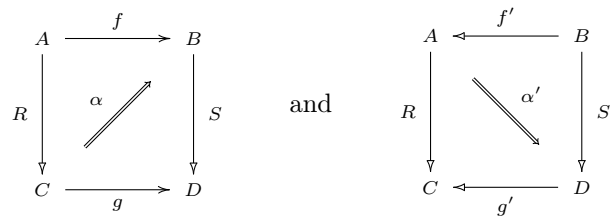
(b) There exist a natural isomorphism



Exercise 3 [15 POINTS]

Australian “mate calculus”: consider a 2-category \mathcal{B} , where the 1-cells are denoted by arrows of the form \rightarrow , while the left-adjoint 1-cells or *maps* are denoted by arrows of the form \dashrightarrow .

Prove that there is a bijective correspondence between 2-cells of the following types:



where $f \dashv f'$ and $g \dashv g'$.