Technische Universität Braunschweig
Institut für Theoretische Informatik
Dr. Jürgen Koslowski

Algebraic Automata Theory

Sheet 4, 2017-11-16

Exercise 1 [10 Points]

In ***set*** : Show explicitly that first taking the kernel pair $\langle P, p_0, p_1 \rangle$ of a function $A \xrightarrow{f} B$ and then taking the coequalizer of p_0 and p_1 up to isomorphism produces the canonical map from A to the set of equivalence classes induced by f .

Exercise 2 [15 Points]

In ***mon*** : Construct explicitly the coequalizer of two parallel monoid homomorphisms, and similarly the pushout of two monoid homomorphisms with common domain. What can you say about the resulting monoid, if the monoids in the original domains are finite?

Exercise 3 [12 Points]

(a) Show that pullbacks as commutative squares are closed under horizontal and vertical composition.

(b) Conversely, if the, say, horizontal composition of two commutative squares is a pullback, what can you say about the individual squares?

(c) Try to characterize monos by means of their kernel pairs.

Exercise 4 [8 Points]

Show the equivalence to the following definitions of congruences E on a monoid $\langle M, \cdot, e \rangle$:

(a) $E \subseteq M \times M$ is a sub-monoid of the cartesian product, *i.e.*, $\langle a, b \rangle, \langle c, d \rangle \in E$ implies $\langle a, b \rangle \cdot \langle c, d \rangle = \langle a \cdot c, b \cdot d \rangle \in E$.

(b) $\langle a, b \rangle \in E$ implies that for all $u, v \in M$ we have $\langle u \cdot a \cdot v \, u \cdot b \cdot v \rangle \in E$.

due on Thursday, 2017-11-23, 13:15,