Correction on pre*:

Problem: ↳ The algorithm only works correctly in the sense that

$$CF(A_{pre^*}) = pre^*(CF(A))$$

<u>if the initial states of the P-NFA A do not have incoming arcs.</u>

↳ This can always be achieved by unwinding loops (see below).

A flawed example:

↳ Consider PDS $P =$ 

↳ Consider P-NFA $A =$ 

that represents the set of configurations

$$CF(A) = \{(q, \beta^k) \mid k \in \mathbb{N}\}$$

↳ The P-NFA does not obey the constraint on initial states (no incoming arcs) stated above.

↳ In fact, pre* yields an incorrect result. We have

$A_{pre^*} =$ 

But

$$CF(A_{pre^*}) \not\subseteq pre^*(CF(A))$$

and thus

$$CF(A_{pre^*}) \neq pre^*(CF(A)).$$

To check that the inclusion fails, consider

$$(q, \beta.\alpha) \in CF(A_{pre^*}).$$

We have

$(q, \beta.\alpha) \not\to^* c$ with $c \in CF(\mathcal{A}) = \{(q, \beta^k) \mid k \in \mathbb{N}\}$.

- The reason is that no transition of P is able to remove β.
- But $(q, \beta.\alpha)$ itself is not in $CF(\mathcal{A})$.

To fix the problem, we adopt the definition of P-NFAs (and exclude those that let the algorithm fail.)

<u>Definition (P-NFA):</u>

Let $P = (Q, \Gamma, \to)$ a PDS.

A <u>P-NFA</u> is an NFA

$$\mathcal{A} = (S, S_I, \to, S_F) \text{ over } \Gamma$$

where

- $S_I = \{s_q \mid q \in Q\}$
- <u>there is no transition leading to an initial state</u>.

The remainder of the pre* construction is left unchanged.

<u>Example:</u>

- To represent $\{(q, \beta^k) \mid k \in \mathbb{N}\}$ in the example above, use the P-NFA

$\mathcal{A} =$

When we now apply pre*, we obtain

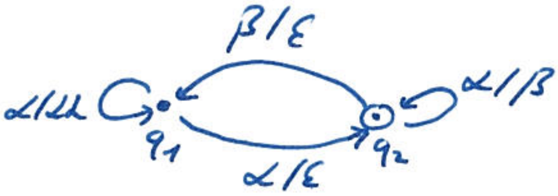
$\mathcal{A}_{pre^*} =$

which is correct: $CF(\mathcal{A}_{pre^*}) = pre^*(CF(\mathcal{A}))$.

Correction of the BPDS example from the lecture:

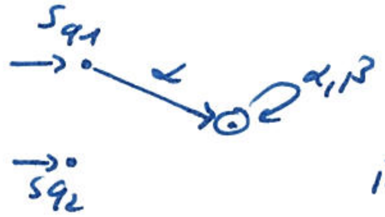
Consider

BP



- Determine $pre^*(\{q_1\} \times \alpha.\Gamma^*)$

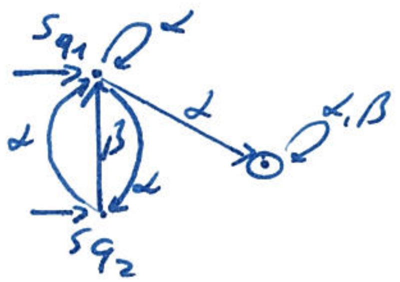
where $\{q_1\} \times \alpha.\Gamma^*$ is represented by



A

Then

A_{pre^*} =

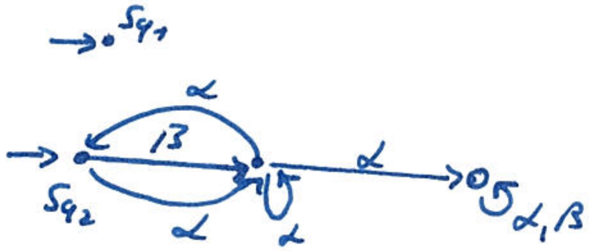


- Intersect $CF(A_{pre^*}) \cap Q_F \times \Gamma^*$

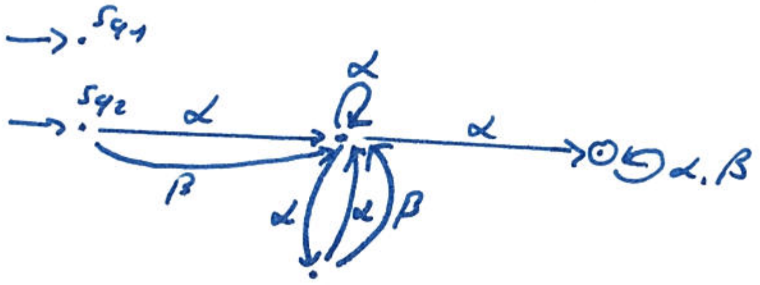
$= CF(A_{pre^*}) \cap \{q_2\} \times \Gamma^*$

$= CF(A')$

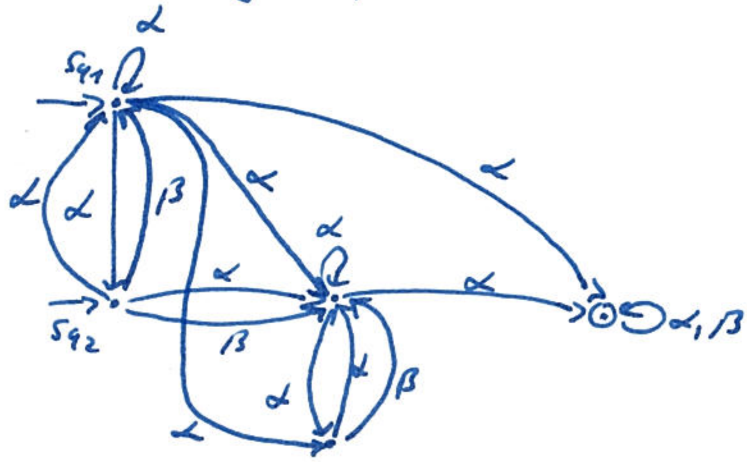
with A' =



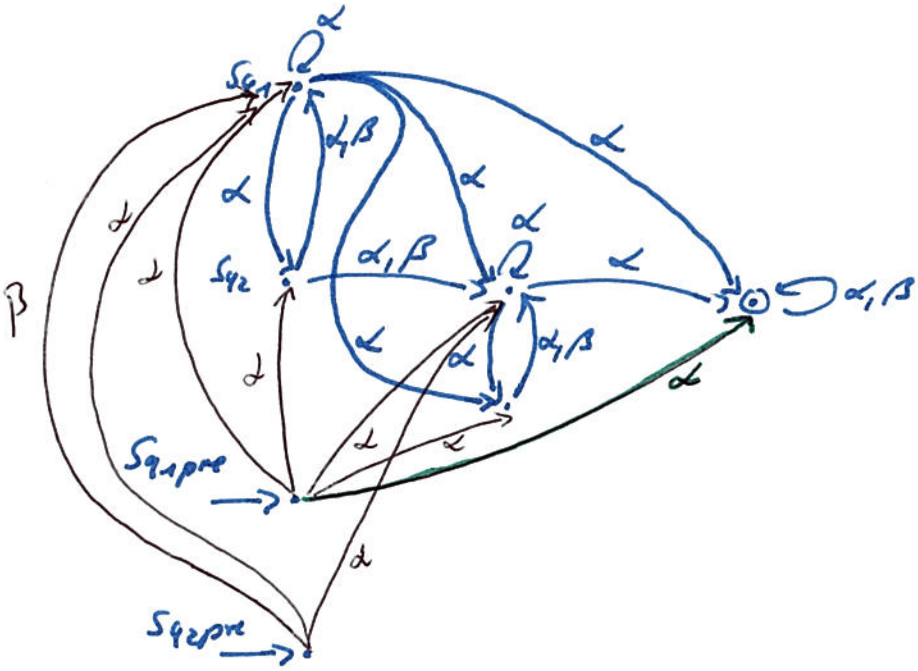
- Use unwinding to construct P-NFA
that has no incoming arcs to initial states
(to construct P-NFA according to new definition):



• Determine again pre*:



• Finally, compute a single pre:



<u>The side condition was not needed for the proofs in the lecture</u>

↳ So why are the proofs correct but the method only works with the above restriction?

↳ Skipped the proof of the technical lemma, the restriction is used there.