Advanced Automata Theory
Exercise Sheet 1

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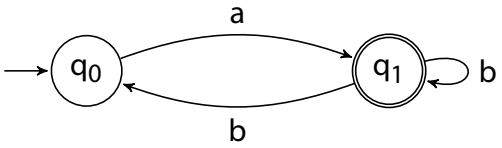
Out: 21 April

Due: 25 April, 12:00

Submit your solutions until Monday, 12:00, in the box in the staircase of building 34, 4^{th} floor.

Exercise 1: REG $\Rightarrow$ NFA

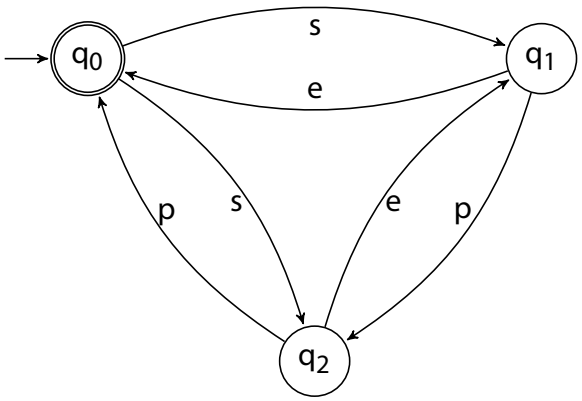
Use the methods discussed in the lecture to prove that $ab^+ \subseteq \mathcal{L}(A)$, where the NFA A is specified as follows:



Hint: Transform the regular expression into an NFA.

Exercise 2: NFA $\Rightarrow$ REG

Use Arden's Lemma to find a regular expression for $\mathcal{L}(A)$, where A is specified as follows:



Exercise 3: Arden's Lemma

Consider the following extension of Arden's Lemma:
If $U, V \subseteq \Sigma^*$ and $\varepsilon \in U$ then all solutions $L \subseteq \Sigma^*$ of the equation $L = UL \cup V$ are precisely the elements of $\mathcal{L} = \{U^*V' \mid V \subseteq V' \subseteq \Sigma^*\}$.

Prove the extension by solving a) and b) below:

a) Show that if L is a solution of $L = UL \cup V$ then $L \in \mathcal{L}$.

b) Show that every $L \in \mathcal{L}$ satisfies $L = UL \cup V$.

Exercise 4: Languages & Formulas

Provide some arguments with your solution for the following tasks:

a) Find a formula φ such that $L(\varphi) = \Sigma^* a \Sigma^* b^+$.

b) What is the language described by $\exists y \forall x \forall z\colon (x < y \wedge y < z) \rightarrow \neg P_a(x) \wedge P_b(y)$?