Advanced Automata Theory
Exercise Sheet 3

Emanuele D'Osualdo
Sebastian Muskalla
TU Kaiserslautern
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Exercise 1: Ehrenfeucht-Fraïssé Games

Let $n \in \mathbb{N}$ be arbitrary. Which is the maximal number of rounds $k \in \mathbb{N}$ such that the duplicator has a winning strategy for $G_k\left((ab)^{2n+1}, (ba)^{2n+1}\right)$?

Hint: First see what happens for $n = 1$ and $n = 2$.

Exercise 2: More Ehrenfeucht-Fräísse Games

Let $n \in \mathbb{N}$ be arbitrary. For which k does the Duplicator win $G_k\left(a^n b a^n, a^n b a^{n+1}\right)$?

Exercise 3: Star-Free Languages

Prove or disprove whether the following languages over $\Sigma = \{a, b\}$ are star-free:

a) $(ab \cup ba)^*$

b) $(a \cup bab)^*$

c) $\mathcal{L}_{odd} = \{w \in \Sigma^* \mid w \text{ has odd length}\}$

Exercise 4: Star-Free $\Rightarrow$ FO[<]-definable

a) Let $w = a_0 \dots a_n \in \Sigma^*$ be a word and let $i, j \in \mathbb{N}$ such that $0 \leq i \leq j \leq n$. Show that for every closed FO[<]-formula φ and FO-variables x, y with $\mathcal{I}(x) = i, \mathcal{I}(y) = j$, there is a formula $\psi(x, y)$ such that

$$\mathcal{S}(w), \mathcal{I} \vDash \psi \text{ if and only if } \mathcal{S}(a_i \dots a_j) \vDash \varphi.$$

b) Deduce from a) that FO[<]-definable languages are closed under concatenation.

c) Prove using structural induction that every star-free language is FO[<]-definable.