	Advanced Automata Theory	
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Out: May 25, **Updated May 26:** Fixed bug in counter machine Ex. 1b   Due: May 30, 12:00

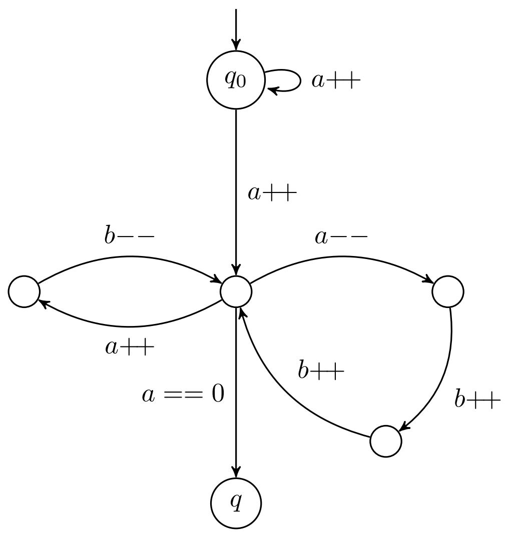
Exercise 1: Reversal-Bounded Counter Machines

Consider an n-counters machine where all the counters are unrestricted. The set of reachable vectors at a state q of a counter machine is the set $R(q) := \{\vec{v} \in \mathbb{N}^n \mid (q_0, 0) \to^* (q, \vec{v})\}$. The set of reachable vectors at a state q using at most k reversals is

$$R_k(q) := \left\{ \vec{v} \in \mathbb{N}^n \;\middle|\; (q_0, 0) \xrightarrow[\leq k \text{ reversals}]{}^* (q, \vec{v}) \right\}.$$

a) Prove that $R_k(q)$ is semilinear.

b) Consider the following counter machine:



Show that $R(q) \supsetneq R_1(q)$.

c) Compute a semilinear set representing $R_1(q)$ for the machine above.

Exercise 2: Naive Interpretation of NFAs as NBAs

Let $A = (\Sigma, Q, q_0, \to, Q_F)$ be an NFA with $\emptyset \neq L(A) \subseteq \Sigma^+$ and, for any two states $q, q' \in Q$, define $L^{\neq\epsilon}_{q,q'} := \{w \in \Sigma^+ \mid q \xrightarrow{w} q' \text{ in } A\}$. If $L_\omega(A)$ is the ω-regular language accepted by A (interpreted as an NBA), one can **wrongly** believe that $L_\omega(A) = L(A)^\omega$.

a) Find a counterexample to $L_\omega(A) = L(A)^\omega$ when $\emptyset \neq L^{\neq\epsilon}_{q,q} \subseteq L(A)$ for all $q \in Q_F$.

b) ~~Show that if $L(A) = L^+$ for some regular language L, then $L_\omega(A) = L(A)^\omega$ holds.~~
Update: That actually does not hold...

c) Given an NFA A, provide a construction for an NBA A_ω such that $L(A_\omega) = L(A)^\omega$.

Exercise 3: NBA languages = ω-regular Languages

a) Prove that ω-regular languages are NBA definable.

b) Show that if there exists an NBA that accepts $L \subseteq \Sigma^\omega$ then L is ω-regular.

c) Construct an NBA that accepts $L = (ab + c)^*((aa + b)c)^\omega + (a^*c)^\omega$.

Exercise 4: Shuffle ω-regular Languages

Given an infinite set of positions $I \subseteq \{0, 1, \ldots\}$ with $I = \{i_1, i_2, \ldots\}$ and $i_1 < i_2 < \ldots$, and an ω-word w, we write $w|_I$ for the ω-word $w(i_1)w(i_2)\ldots$, i.e. the sub-word of w obtained by selecting the letters in the positions of I.

The **fair shuffle** of two ω-languages L_1, L_2 is defined as

$$L_1 \sqcup L_2 := \{w \mid \exists \text{ partition } I, J \text{ of positions } \{0, 1, \ldots\} \text{ such that } w|_I \in L_1 \text{ and } w|_J \in L_2\}$$

Note in particular, that since I and J form a partition of the positions, $I \neq \emptyset \neq J$.

Show that ω-regular languages are closed under fair shuffle.

Exercise 5: Reachability in Counter Machines (Optional)

Adapting Parikh's proof, show that reachability in counter machines with one unrestricted counter and n r-reversal bounded counters is decidable.