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Advanced Automata Theory
Exercise Sheet 7

Summer term 2016

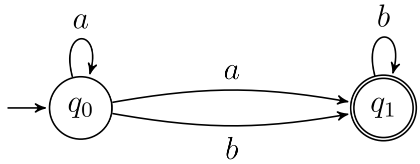
Out: June 1 Due: June 6, 12:00

Exercise 1: Variation of Ramsey's Theorem

Let (V, E) be an infinite graph such that for every infinite set of vertices $X \subseteq V$ there are $v, v' \in X$ with $(v, v') \in E$. Prove that (V, E) contains an infinite complete subgraph.

Exercise 2: NBA Complementation

Consider the NBA A over $\Sigma = \{a, b\}$ below:



Use Büchi's complementation method discussed in class to compute $L(A)$ and $\overline{L(A)}$.

Exercise 3: Equivalence

Consider an NBA A, two classes $[u]_{\sim_A}$ and $[v]_{\sim_A}$ of $\sim_A$, and $w \in [u]_{\sim A} \cdot [v]^{\omega}_{\sim A}$ an ω-word. Show that if $w \in L(A)$ then $[u]_{\sim A} \cdot [v]^{\omega}_{\sim A} \subseteq L(A)$.

Exercise 4: Muller Automata

A Nondeterministic Muller Automaton (NMA) is a tuple $A = (Q, \Sigma, \delta, q_0, F)$. The first four components are as in Büchi automata. $F = \{Q_F^1, \ldots, Q_F^n\} \subseteq \mathcal{P}(Q)$ is a set of sets of states instead of a single set of states. The idea is to accept a run if the set of states that occur infinitely often matches one of the Q_F^i exactly. Formally, a run r of A is accepting if $\mathit{Inf}(r) \in F$ where $\mathit{Inf}(r)$ is the set of states that are visited infinitely often in r. As for Büchi automata, we call A a Deterministic Muller Automaton (DMA) if for each $q \in Q$ and $a \in \Sigma$ there is exactly one state $q' \in Q$ such that $(q, a, q') \in \delta$.

a) Given an NBA A, show that there is an NMA A_{NMA} such that $L(A_{NMA}) = L(A)$.

b) Show that DMA are strictly more expressive than DBA.

c) Given a DMA A, show that there is an NBA A_{NBA} such that $L(A_{NBA}) = L(A)$.

d) Prove that DMA are closed under complement, i.e. for every DMA A there exists a DMA $\bar{A}$ with $L(\bar{A}) = \overline{L(A)}$.