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Advanced Automata Theory
Exercise Sheet 8

TU Kaiserslautern
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Out: June 9 Due: June 13, 12:00

Exercise 1: NBA and S1S

MSO[$<$, succ] is the monadic second order logic interpreted on ω-words in the expected way. Its (clearly equiexpressive) fragment MSO[succ] is commonly known as S1S, the (monadic) second order logic of one successor.

a) Define a S1S formula Inf(X) so that $S_w, I \models \text{Inf(X)}$ iff $I(X)$ is an infinite set.

b) Büchi's theorem (I) can be adapted to show that every NBA-definable language is S1S-definable. Illustrate the main ingredients needed to adapt Büchi's proof.

c) Büchi's theorem (II) can be adapted to show that every S1S-definable language is NBA-definable. Illustrate the main ingredients needed to adapt Büchi's proof.

Exercise 2: LTL

a) Show that every LTL-definable language is FO[$<$]-definable.[1]

b) EF-games and the EF-theorem remain valid for ω-languages too.
Making use of this fact, show that $(a\,\{a,b\})^\omega$ is not LTL-definable.

c) Recall that the regular language $(aa)^*$ is **not** FO-definable. Why do we need at least two letters in the alphabet, to separate FO in the ω-languages case?

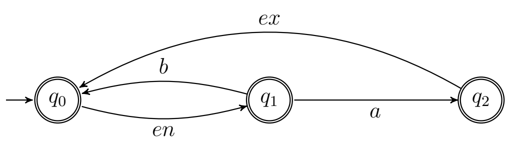
Exercise 3: Fairness

We define three notions of fairness (*en* and *ex* stand for "enabled" and "executed"):

Absolute fairness (impartiality):	$\Box\Diamond\, ex$	(**AF**)
Strong fairness (compassion):	$\Box\Diamond\, en \to \Box\Diamond\, ex$	(**SF**)
Weak fairness (justice):	$\Diamond\Box\, en \to \Box\Diamond\, ex$	(**WF**)

Which of the following statements hold for the NBA A depicted below?

$A \models \mathbf{AF} \to \Box\Diamond\, a$ $\quad$ $A \models \mathbf{SF} \to \Box\Diamond\, a$ $\quad$ $A \models \mathbf{WF} \to \Box\Diamond\, a$



[1]here FO[$<$] is the first order fragment of MSO[$<$] over ω-words

Exercise 4: Unrollings

Prove the following equivalences:

(a) $\varphi \,\mathcal{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathcal{U}\, \psi))$

(b) $\varphi \,\mathcal{R}\, \psi \equiv \psi \wedge (\varphi \vee \bigcirc(\varphi \,\mathcal{R}\, \psi))$