Advanced Automata Theory
Exercise Sheet 11

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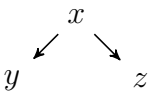
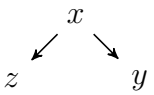
Exercise 1: Path Closure

Let $t\colon T \to \Sigma$ be a Σ-tree with maximum rank k and $D_k = \{0, \dots, k-1\}$.
The **path language** $\pi(t) \subseteq (\Sigma \cup D_k)^*$ is defined inductively by

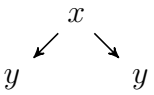
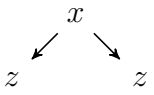
- if $\mathrm{rk}(t(\epsilon)) = 0$ *(i.e. t is a leaf)*, then $\pi(t) := \{t(\epsilon)\}$
- if t has subtrees $t_0, \dots, t_n$, then $\pi(t) := \bigcup_{i=0}^{n} \{t(\epsilon) \cdot i \cdot w \mid w \in \pi(t_i)\}$

For a tree language L, we define $\Pi(L) := \bigcup_{t \in L} \pi(t)$.
The **path closure** of L is $pc(L) := \{t \mid \pi(t) \subseteq \Pi(L)\}$.
L is **path-closed** if and only if $L = pc(L)$.

Example:

$t_1 =$  and $t_2 =$ 

Then $\pi(t_1) = \{x0y, x1z\}$, $\pi(t_2) = \{x0z, x1y\}$ and $\Pi(\{t_1, t_2\}) = \{x0y, x1z, x0z, x1y\}$. The path closure $pc(\{t_1, t_2\})$ contains t_1, t_2 and additionally the trees

$t_3 =$  and $t_4 =$ 

(a) Prove that if L is a regular tree language, then $\pi(L)$ is a regular language.

(b) Prove that if L is a regular tree language, then also $pc(L)$ is a regular tree language.

(c) Let L be a regular tree language. Prove that L is path-closed if and only if L is recognizable by a DTDTA.

(d) Is it decidable whether a regular tree language defined by a BUTA is path-closed?

Exercise 2: Finiteness of BUTA Tree Language

Show that it is decidable whether the tree language accepted by a given BUTA is finite.
Hint: search for loops

Exercise 3: Boolean Satisfiability

Let $\Sigma = \{\wedge/_2, \vee/_2, \neg/_1, 0/_0, 1/_0, x/_0\}$.

(a) Give a deterministic BUTA that recognises the satisfiable Boolean formulas over x.

(b) Use (a) to establish whether $\neg(x \wedge 0) \vee (\neg x \wedge 1)$ and $(x \vee 0) \wedge (\neg x \wedge 1)$ are satisfiable.

Exercise 4: Tree Language Acceptance

Let $\Sigma = \{a/_2, b/_2, c/_0, d/_0\}$. Establish which of the following tree languages are accepted by some BUTA.

(a) $L_1 := \{t \in \mathcal{T}_\Sigma \mid \text{the path } \epsilon, 0, 01, 010, 0101, \ldots \text{ in } t \text{ contains an even number of } a\text{'s}\}$.

(b) $L_2 := \{t \in \mathcal{T}_\Sigma \mid t \text{ is an unbalanced tree}\}$.

(c) $L_3 := \{t \in \mathcal{T}_\Sigma \mid \text{there are nodes } u, v \text{ in } t \text{ with } t(u) = c,\ t(v) = d \text{ and } u \text{ is left of } v\}$.

(d) $L_4 := \{t \in \mathcal{T}_\Sigma \mid \text{precisely 2016 of } t\text{'s leaves are labelled by } c\}$.

Determine which of the languages above are also accepted by a deterministic TDTA.