Exercise 1: Parity Game Determinacy

Use the McNZSolver procedure from class to solve both parity games shown below.
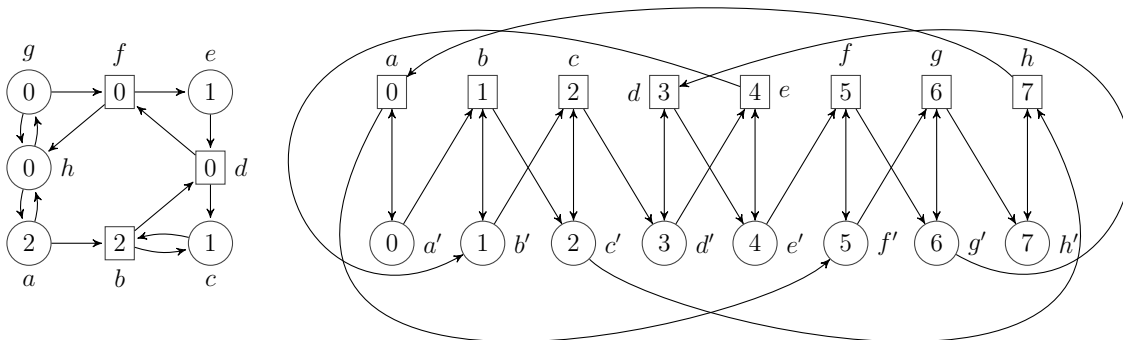
Exercise 2: PTA Emptiness

Let $A = ((\Sigma, rk), Q, q_0, \rightarrow, \Omega)$ be a PTA with maximal rank $n \in \mathbb{N}$ in Σ.

We define $\Sigma^- := \{a\}$ with $rk^-(a) = n$.

Then we set $A^- := ((\Sigma^-, rk^-), Q, q_0, \rightarrow^-, \Omega)$ with

$$q \rightarrow_a^- (q^0, \ldots, \underbrace{q^i, q^i, \ldots, q^i}_{n-i+1 \text{ times}}) \text{ if } \exists b \in \Sigma \text{ with } rk(b) = i+1 \text{ and } q \rightarrow_b (q^0, \ldots, q^i)$$

Show that $L(A^-) = \emptyset$ iff $L(A) = \emptyset$.

Remark: this tells you that if you are only interested in emptiness of $L(A)$ you do not need to care about the alphabet symbols, but just about existence of runs. The annoying detail is that you then need to make the rank uniform as well.

Exercise 3: PTA Acceptance as a Parity Game

Let $A = (\{a/_2, b/_2\}, \{q_0, q_1\}, q_0, \rightarrow, \Omega)$ be a PTA with $\Omega(q_0) = 1$, $\Omega(q_1) = 2$, and

$$q_0 \rightarrow_a (q_0, q_0) \qquad q_0 \rightarrow_b (q_1, q_1) \qquad q_1 \rightarrow_a (q_0, q_0) \qquad q_1 \rightarrow_a (q_1, q_1) \qquad q_1 \rightarrow_b (q_1, q_1).$$

(a) What is $L(A)$ and $\overline{L(A)}$?

(b) Given a tree $t_1 \in L(A)$, show a positional winning strategy for player A in $G(A, t_1)$.

(c) Given a tree $t_2 \notin L(A)$, show a positional winning strategy for player P in $G(A, t_2)$.

(d) Let $t \notin L(A)$ be the following tree
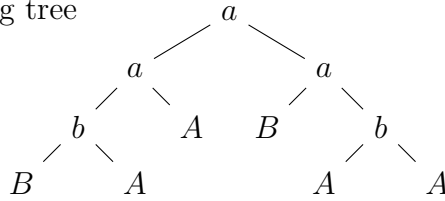
where $A = a(A, A)$ and $B = b(B, B)$, i.e. A is an infinite tree consisting of only a's and B is an infinite tree of b's.

Use the construction presented in class to decorate the tree with a strategy for player P.

Exercise 4: Complementation of Deterministic PTA

(a) Given a deterministic PTA A, construct a deterministic PTA A' with $L(A') = \overline{L(A)}$.

(b) Does your construction also work for the nondeterministic case, and why?

Exercise 5: Exam Appointments

Please contact Sebastian (`schweizer@cs.uni-kl.de`) to get an appointment for the exam.