

Advanced Automata Theory

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Exercise Sheet 3

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Due: May 2, 09:30

Exercise 1: Star-Free Languages

Prove or disprove whether the following languages over $\Sigma = \{a, b\}$ are star-free:

- a) $(ab \cup ba)^*$
- b) $(a \cup bab)^*$
- c) $\mathcal{L}_{\text{odd}} = \{w \in \Sigma^* \mid w \text{ has odd length}\}$

Exercise 2: Star-Free \Rightarrow FO[<]-definable

- a) Let $w = a_0 \dots a_n \in \Sigma^*$ be a word and let $i, j \in \mathbb{N}$ such that $0 \leq i \leq j \leq n$. Show that for every closed FO[<]-formula φ and FO-variables x, y with $\mathcal{I}(x) = i, \mathcal{I}(y) = j$, there is a formula $\psi(x, y)$ such that

$$\mathcal{S}(w), \mathcal{I} \models \psi \text{ if and only if } \mathcal{S}(a_i \dots a_j) \models \varphi.$$

- b) Deduce from a) that FO[<]-definable languages are closed under concatenation.
- c) Prove using structural induction that every star-free language is FO[<]-definable.