SS 2018 25.04.2018

Exercises to the lecture
Algorithmic Automata Theory
Sheet 2

Dr. Prakash Saivasan
Peter Chini Delivery until 02.05.2018 at 09:00

Exercise 2.1 (WMSO to Finite Automata)
Using the method presented in the lecture, construct a finite automaton that accepts the language defined by the formula

$$\varphi = \exists x \exists y \colon x < y \land P_a(x) \land P_a(y) \ .$$

Exercise 2.2 (WMSO Expressiveness)

a) Show that WMSO[$<$, suc] and WMSO[suc] are equally expressive.

b) Show that WMSO[$<$, suc] and WMSO[$<$] are equally expressive.

Exercise 2.3 (Star-Free Languages)
Prove or disprove whether the following languages over $\Sigma = \{a, b\}$ are star-free:

a) $(ab \cup ba)^*$

b) $(a \cup bab)^*$

c) $L_{odd} = \{w \in \Sigma^* \mid w \text{ has odd length}\}$.

Exercise 2.4 (Star-Free $\Rightarrow$ FO[$<$]-definable)

a) Let $w = a_0 \dots a_n \in \Sigma^*$ be a word and let $i, j \in \mathbb{N}$ such that $0 \leq i \leq j \leq n$. Show that for every closed FO[$<$]-formula φ and FO-variables x, y with $\mathcal{I}(x) = i, \mathcal{I}(y) = j$, there is a formula $\psi(x, y)$ such that
$$\mathcal{S}(w), \mathcal{I} \models \psi \text{ if and only if } \mathcal{S}(a_i \dots a_j) \models \varphi.$$

b) Deduce from a) that FO[$<$]-definable languages are closed under concatenation.

c) Prove using structural induction that every star-free language is FO[$<$]-definable.

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