SS 2018 28.05.2018

Exercises to the lecture
Algorithmic Automata Theory
Sheet 6

Dr. Prakash Saivasan
Peter Chini Delivery until 12.06.2018 at 12:00

Exercise 6.1 (Quantifier Elimination)
Eliminate the quantifiers in the following formula with the method described in class.

$$\varphi = \neg\forall x : 3x < 2y \vee y < 2x$$

Exercise 6.2 (Parikh Image of Regular Languages)

a) Let L be a regular language. Show that the Parikh Image $\Psi(L)$ is semi-linear.

b) Show that for each semi-linear set $S \subseteq \mathbb{N}^d$, there is a regular language L over $\Sigma = \{a_1, \ldots, a_d\}$ with $S = \Psi(L)$.

Exercise 6.3 (Closure Properties of Semi-linear Sets I)
Let $S = \bigcup_{i=1}^{\ell} L(c_i, P_i) \subseteq \mathbb{N}^n$ be semi-linear.

a) Show that S^* can be described as:

$$S^* = \bigcup_{I \subseteq \{1,\ldots,\ell\}} L\left(\sum_{i \in I} c_i, \bigcup_{i \in I} P_i \cup \{c_i\}\right).$$

b) Let $f : \mathbb{N}^n \to \mathbb{N}^m$ be a linear function. Prove that $f(S)$ is semi-linear.

Exercise 6.4 (Closure Properties of Semi-linear Sets)
To prove that semi-linear sets are closed under intersection, we showed that the intersection of two linear sets $L(c, \{u_1, \ldots, u_m\})$ and $L(d, \{v_1, \ldots, v_n\}) \subseteq \mathbb{N}^d$ is semi-linear. We defined

$$B = \left\{ (x_1, \ldots, x_m, y_1, \ldots, y_n) \in \mathbb{N}^{m+n} \,\middle|\, \sum_{i=1}^{m} x_i \cdot u_i = \sum_{i=1}^{n} y_i \cdot v_i \right\}.$$

Let S_B be the set of minimal elements with respect to the product order: We have $u \leq v$ if and only if $u(i) \leq v(i)$ for all $i = 1, \ldots, d$.
Prove that $B = L(0, S_B)$.

Delivery until 12.06.2018 at 12:00 into the box next to 343