Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 4

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Exercise 4.1

Show that the following languages are star-free:

(a) $\{a\}^*\{b\}^*$,

(b) the set of words $w \in \{a, b\}^*$ that contain the subword ab as often as the subword ba.

Exercise 4.2

Given an automaton A and a WMSO[$<$, suc]-formula φ, the *model checking* problem asks whether every word accepted by A satisfies φ. If yes, we write $A \models \varphi$.

Show that the model checking problem is Turing reducible to the problem of whether in a finite automaton, one given state can be reached.

Exercise 4.3

Let LTL (linear time logic) formulas be defined by the grammar

$$\varphi \quad ::= \quad \text{true} \quad | \quad a \quad | \quad \varphi_1 \wedge \varphi_2 \quad | \quad \neg\varphi \quad | \quad \bigcirc \varphi \quad | \quad \varphi_1 \, \mathbf{U} \, \varphi_2 \quad | \quad \Diamond\varphi \quad | \quad \Box\varphi,$$

where $a \in \Sigma$ are possible *actions* of a system, e.g. *request*, *response*, etc.. Intuitively, an LTL formula describes properties of action sequences, when they are consumed from left to right. The formulas $\bigcirc\varphi$, $\varphi_1 \, \mathbf{U} \, \varphi_2$, $\Diamond\varphi$, and $\Box\varphi$ mean "*next* φ", "φ_1 *until* φ_2", "*eventually* φ", and "*globally* φ", respectively. LTL formulas are commonly used to describe temporal properties of systems, for example

$\Box\,(\neg ack \; \mathbf{U} \; req)$:	There is no acknowledge before a request.
$\Box\,(req \rightarrow \Diamond ack)$:	Every request is followed by an acknowledge.
$\Box\,(req \rightarrow \bigcirc ack)$:	Every request is immediately followed by an acknowledge.

Give a recursive procedure which transforms every LTL formula into a corresponding FO[$<$, suc] formula. *Hint*: if you can express $\Diamond$ and $\Box$ by the other LTL constructs, you do not have to translate them.

Exercise 4.4

Construct a finite automaton for the Presburger formula $\exists y.\, x = 3y$.