Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 5

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Exercise 5.1 Quantifier Elimination

Eliminate the quantifiers of the following formulas using the method described in class:

(a) $\exists x.\, u - 1 < 3x \wedge 2x < t + 6 \wedge x \equiv_2 1$

(b) $\exists x.\, [x + u \geq 0 \vee x \equiv_5 2] \rightarrow [3x + t \leq 1 \wedge 1 \leq 2x - u]$

Exercise 5.2 Semilinear Sets

(a) Prove that for a semilinear set $S \subseteq \mathbb{N}^n$ and $v \in \mathbb{N}^n$, it is decidable whether $v \in S$.

(b) Prove that $S_1 + S_2 := \{x + y \mid x \in S_1, y \in S_2\}$ is semilinear if S_1, S_2 are semilinear.

Exercise 5.3 Parikh Images of Regular Languages

For a word $w \in \Sigma^*$, the *Parikh image* $\Psi(w) : \Sigma \mapsto \mathbb{N}$ yields the number of occurrences of each letter in w: $\Psi(w)(a) = \sharp_a$ in w. For example, $\Psi(aabbb)(a) = 2$ and $\Psi(aabbb)(b) = 3$. Thus, $\Psi(w) \in \mathbb{N}^{|\Sigma|}$ is a vector and for a language L, we define $\Psi(L) := \{\Psi(w) \mid w \in L\}$.

(a) Prove that $\Psi(L)$ is semilinear if $L \in \mathrm{REG}_\Sigma$. (Prove the equalities you use as well.)

(b) Prove that for a semilinear set $S \subseteq \mathbb{N}^n$ there is a regular language L with $S = \Psi(L)$.

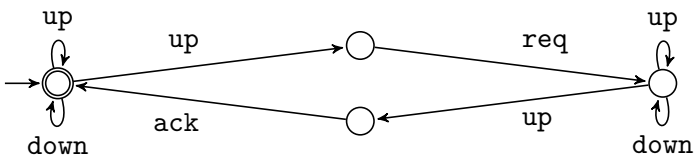
Exercise 5.4 Semilinear Extensions of Regular Languages

Consider extended regular expressions (r, S) where $r \in \mathrm{REG}_\Sigma$ and $S \subseteq \mathbb{N}^{|\Sigma|}$ is semilinear, and define $L(r, S) := \{w \in \Sigma^* \mid w \in L(r) \text{ and } \Psi(w) \in S\}$.

(a) Prove that emptiness of $L(r, S)$ is decidable for any extended regexp (r, S).

(b) Find an extended regexp (r, S) such that $a^n b^n c^n = L(r, S)$.

(c) What is the language r accepted by the "request/acknowledge" automaton below?



Describe the semilinear set S for which the extended regexp $L(r, S)$ represents $> 80\%$ system availability, i.e. the transition sequences with $\leq 20\%$ `down` time.