Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 8

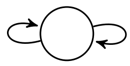
Jun.-Prof. Roland Meyer, Reiner Hüchting, Georgel Călin Due: Tue, Dec 11 **(noon)**

Exercise 8.1 Disjunctive Well-Foundedness

Consider the following program over integer variables and the corresponding automaton:

while $x > 0 \wedge y > 0$ **do**
  $\texttt{l}_\texttt{a}$: $(\texttt{x}, \texttt{y}) := (\texttt{x} - 1, \texttt{x})$
or
  $\texttt{l}_\texttt{b}$: $(\texttt{x}, \texttt{y}) := (\texttt{y} - 2, \texttt{x} + 1)$
endwhile

l_a : **if** $x > 0 \wedge y > 0$
$\texttt{x}' := \texttt{x} - 1$
$\texttt{y}' := \texttt{x}$



l_b : **if** $x > 0 \wedge y > 0$
$\texttt{x}' := \texttt{y} - 2$
$\texttt{y}' := \texttt{x} + 1$

A state S of this program is a vector giving a value to each variable. The execution of a command l_a or l_b leads to a labelled transition between states. For example:

$$S = (x = 2, y = 1) \xrightarrow{l_a} (1, 2) = S'.$$

One can show that between every pair of states $S \xrightarrow{w} S'$, where $w \in \{l_a, l_b\}^+$, one of the following relations holds:

T_1	$x > 0 \wedge x > x'$
T_2	$x + y > 0 \wedge x + y > x' + y'$
T_3	$y > 0 \wedge y > y'$

Show that this implies termination (from any starting state).

Exercise 8.2 Equivalence Classes as Circuit Boxes

Remember that, for any $u \in \Sigma^\omega$ and NBA A, $\text{Box}(u)$ is defined as $R_{[u]_{\sim_A}} \cup R^{\text{fin}}_{[u]_{\sim_A}}$.

(a) Prove that $[u]_{\sim_A} = [v]_{\sim_A}$ if and only if $\text{Box}(u) = \text{Box}(v)$.

(b) Prove that $\text{Box}(uv) = \text{Box}(u) \,;\, \text{Box}(v)$, where ";" glues boxes together.

(c) Give an algorithm in pseudo code which computes all $\sim_A$ equivalence classes (boxes).

Exercise 8.3 NBA Emptiness and Membership

Let A be an NBA and uv^ω be an ω-word. Give algorithms that decide whether:

$$L(A) = \emptyset \qquad\qquad uv^\omega \in L(A).$$

Exercise 8.4 NBA Complementation

Compute $L(A)$ and $\overline{L(A)}$ for the NBA A below: