Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 9

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Exercise 9.1 NBA Inclusion

Give NBAs A, B over $\{a, b\}$ with $L(A) = ba^\omega$ and $L(B) = (a+b)^* a^\omega$. Construct another NBA C – as explained in class – such that $L(A) \subseteq L(B)$ precisely when $L(C) = \Sigma^\omega$.

Exercise 9.2 LTL Laws

Establish whether the following congruences hold or do not hold:

(a) $\Diamond\Diamond\varphi \equiv \Diamond\varphi$
(b) $\Box\Box\varphi \equiv \Box\varphi$
(c) $\Box\Diamond\Box\varphi \equiv \Diamond\Box\varphi$
(d) $\Diamond\Box\Diamond\varphi \equiv \Box\Diamond\varphi$
(e) $\bigcirc\Diamond\varphi \equiv \Diamond \bigcirc \varphi$
(f) $\Box\varphi \wedge \bigcirc\Diamond\varphi \equiv \Box\varphi$
(g) $\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$
(h) $\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \wedge \Diamond\psi$
(i) $\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$
(j) $\varphi\, \mathcal{U}\, (\varphi\, \mathcal{U}\, \psi) \equiv \varphi\, \mathcal{U}\, \psi$
(k) $(\Diamond\Box\varphi) \wedge (\Diamond\Box\psi) \equiv \Diamond(\Box\varphi \wedge \Box\psi)$
(l) $\Box\Box(\varphi \vee \neg\psi) \equiv \neg\Diamond(\neg\varphi \wedge \psi)$
(m) $\bigcirc(\varphi\, \mathcal{U}\, \psi) \equiv (\bigcirc\varphi)\, \mathcal{U}\, (\bigcirc\psi)$
(n) $\Box\varphi \rightarrow \Diamond\psi \equiv \varphi\, \mathcal{U}\, (\psi \vee \neg\varphi)$
(o) $(\varphi\, \mathcal{U}\, \psi)\, \mathcal{U}\, \psi \equiv \varphi\, \mathcal{U}\, \psi$

Note: yes/no answer suffice as long as you are able to sustain your claims verbally.

Exercise 9.3 Positive Normal Form

Recall that a formula over $\Sigma = \mathbb{P}(\mathcal{P})$ is in positive normal form (PNF) if expressible by

$$\varphi \quad ::= \quad p \mid \neg p \mid \bigcirc \varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi\, \mathcal{U}\, \varphi \mid \varphi\, \mathcal{R}\, \varphi \quad \text{where } p \in \mathcal{P}.$$

(a) Express $\neg\Big((\Box p) \rightarrow \big((p \wedge \neg r)\, \mathcal{U}\, \neg(\bigcirc q)\big)\Big) \wedge \neg\big(\neg p \vee \bigcirc\Diamond r\big)$ in PNF.

(b) Prove that every LTL formula can be brought to PNF.

Exercise 9.4

(a) In the lecture, LTL was only defined with operators concerning the future. However, it is sometimes convenient to talk about the past as well. Therefore we introduce an operator $\lhd$, where $\lhd\, p$ means "p has held at some time in the past". Express the following formula without $\lhd$:

$$\Box\, (\varphi \rightarrow \lhd\, \psi)$$

(b) We define three notions of fairness (en and ex stand for "enabled" and "executed"):

Absolute fairness (impartiality):	$\Box\Diamond ex$	(**AF**)
Strong fairness (compassion):	$\Box\Diamond en \rightarrow \Box\Diamond ex$	(**SF**)
Weak fairness (justice):	$\Diamond\Box en \rightarrow \Box\Diamond ex$	(**WF**)

Which of the following statements hold for the NBA A depicted below?

$$A \models \mathbf{AF} \to \Box\Diamond a \qquad A \models \mathbf{SF} \to \Box\Diamond a \qquad A \models \mathbf{WF} \to \Box\Diamond a$$