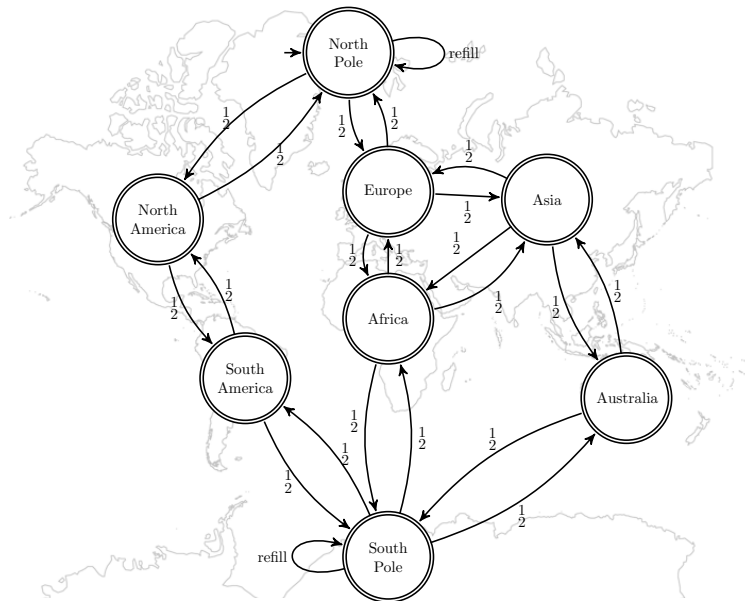


Exercise Sheet 10

Jun.-Prof. Roland Meyer, Reiner Hüchting, Georgel Călin Due: Tue, Jan 08 (**noon**)

Exercise 10.1 Travelling Santa

Santa Claus has recently decided to swap his traditional old fashioned sleigh for the brand new *Christmas Racer 3000* equipped with the most recent state of the art automata driven navigation system *Rudolph Go v0.9beta*. To save weight and increase the fun, Santa wants to carry the least amount of presents possible on each ride. The automaton $A_{Rudolph}$ controlling *Rudolph* is depicted below:



Unfortunately, Santa's first tests have shown that there are still bugs in *Rudolph's* controller. *Rudolph* essentially chooses a random neighbouring continent whenever Santa starts his sleigh. The biggest problem with this is that Santa is seen by one or two children every time he enters a region. Of course he must give a present to every child seeing him. As he can only load new presents into his sleigh when at one of the poles, he possibly runs out of presents. Please help Santa by upgrading *Rudolph* to version 1.0:

- (a) Find out what present capacity the sleigh needs in order not to run out of presents.
- (b) Give an NBA A_{Sleigh} modelling how the number of presents in Santa's sleigh changes.
- (c) Give an LTL formula $\varphi_{Presents}$ describing that Santa never runs out of presents.
- (d) Explain how one can use $A_{Rudolph}$, A_{Sleigh} , and $\varphi_{Presents}$ to create a controller that guarantees Santa never to run out of presents.

- (e) **This exercise is optional.** If you want to be kind to all the children in the world, you modify your controller so that Santa reaches every continent infinitely often.

Exercise 10.2 Tree Decoration

Consider the following rules for decorating a Christmas tree:

φ_1 : Whenever a red ball is added to the tree, a silver ball is added next.

φ_2 : Whenever a candle is added to the tree, a red ball will also be added later.

φ_3 : Whenever a silver ball is added, a candle will be added later and there are no red balls added inbetween.

φ'_3 : Whenever a silver ball is added, a candle is added next.

Translate the above rules into a generalized Büchi automaton:

- (a) Translate the rules into LTL formulas.
- (b) Construct the corresponding GNBA's.
- (c) Give a description of the overall automaton for decorating Christmas trees.

Give this automaton to all your friends and relatives to help them decorate their trees.

Merry Christmas and a Happy New Year!