Applied Automata Theory (WS 2013/2014)	Technische Universität Kaiserslautern

Exercise Sheet 3

Jun.-Prof. Roland Meyer, Georgel Calin Due: Tue, Nov 12

Exercise 3.1 Ehrenfeucht-Fräísse Games

Let $n \in \mathbb{N}$ be arbitrary. For which k does the Duplicator win $G_k(a^n b a^n, a^n b a^{n+1})$?

Exercise 3.2 Star-Free Languages

Prove or disprove whether the following languages are star-free:

(a) $(ab + ba)^*$,

(b) $(a + bab)^*$.

Exercise 3.3 Star-Free ⇒ FO[<]-definable

(a) Let $w = a_0 \dots a_n \in \Sigma^*$ and let $i, j \in \mathbb{N}$ such that $0 \leq i \leq j \leq n$. Show that for every FO[<]-sentence φ and FO-variables x, y with $I(x) = i, I(y) = j$, there is a formula $\psi(x, y)$ such that

$$S(w), I \vDash \psi \text{ if and only if } S(a_i \dots a_j) \vDash \varphi.$$

(b) Deduce from (a) that FO[<]-definable languages are closed under concatenation.

(c) Infer by structural induction that every star-free language is FO[<]-definable.

Exercise 3.4 Presburger Formulas & Parikh Images

(a) Present a Presburger formula ϕ such that every bound variable occurs in precisely one atomic expression and such that

$$Sol(\phi) = \left\{ \begin{pmatrix} 2n+1 \\ n+3 \end{pmatrix} \mid n \in \mathbb{N} \right\} \cup \left\{ \begin{pmatrix} 3n+1 \\ 2n+2 \end{pmatrix} \mid n \in \mathbb{N} \right\}$$

(b) For a word $w \in \Sigma^*$ the *Parikh image* $\Psi(w) : \Sigma \mapsto \mathbb{N}$ yields the number of occurrences of each letter in w. Let $\Psi(L) := \{\Psi(w) \mid w \in L\}$ for any language $L \subseteq \Sigma^*$. Example:

$$\Psi(aabbb) = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \text{ and } \Psi((aa)^*(bbb)^*) = \left\{ \begin{pmatrix} 2n \\ 3p \\ 0 \end{pmatrix} \mid n, p \in \mathbb{N} \right\} \text{ for } \Sigma = \{a, b, c\}.$$

Give an NFA A so that $\Psi(L(A)) = Sol(\phi)$ for the Presburger formula ϕ from (a).