Applied Automata Theory (WS 2013/2014) Technische Universität Kaiserslautern

Exercise Sheet 13

Jun.-Prof. Roland Meyer, Georgel Calin Due: Tue, Feb 4

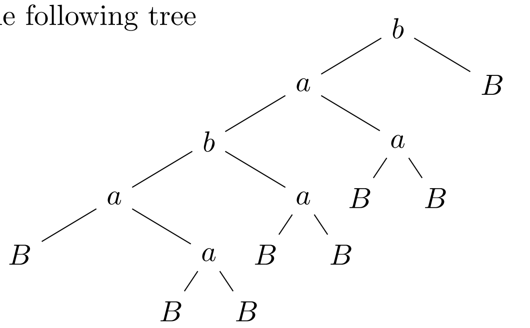
Exercise 13.1 PTA Acceptance as a Parity Game

Let $A = (\{a/2, b/2\}, \{e, o, x\}, e, \rightarrow, \Omega)$ be a PTA with $\Omega(e) = \Omega(o) = 1$, $\Omega(x) = 0$, and

$$\begin{array}{ccccc} e\rightarrow_a(e,o) & e\rightarrow_a(o,e) & e\rightarrow_b(e,e) & e\rightarrow_b(o,o) & e\rightarrow_b(x,x) \\ o\rightarrow_a(e,e) & o\rightarrow_a(o,o) & o\rightarrow_b(e,o) & o\rightarrow_b(o,e) & x\rightarrow_b(x,x) \end{array}$$

(a) What is $L(A)$?

(b) Let $t \notin L(A)$ be the following tree



where $B = b(B, B)$.

Your task: construct a tree t' over $\Sigma \times (\{e, o, x\}^2 \rightarrow \{0, 1\})$ with $\mathsf{proj}_1(t') = t$ and such that $\mathsf{proj}_2(t')$ is a positional winning strategy for the "pathfinder" P in $G(A, t)$.

Exercise 13.2 PTA Complementation

Let $A = (\{a/2, b/2\}, \{q_0, q_1\}, q_0, \rightarrow, \Omega)$ be a PTA with $\Omega(q_0) = 1$, $\Omega(q_1) = 2$, and

$$q_0\rightarrow_a(q_0,q_0) \qquad q_0\rightarrow_b(q_1,q_1) \qquad q_1\rightarrow_a(q_1,q_1) \qquad q_1\rightarrow_b(q_1,q_1).$$

(a) What is $L(A)$?

(b) Argument why the complemented word language L_p of all labeled paths

$$(a_0, s_0, d_0)(a_1, s_1, d_1)\ldots \text{ with } a_i \in \{a, b\},\ s_i \in \{q_0, q_1\}^2 \rightarrow \{0, 1\},\ d_i \in \{0, 1\}$$

from the PTA complementation proof, is precisely the language that satisfies

$$\forall n.\, x_n = a \vee (\exists m.\, x_m = b \wedge \forall n < m.\, x_n = a$$
$$\wedge\ \exists n.(n \leq m \wedge s_n(q_0, q_0) \neq d_n) \vee (n > m \wedge s_n(q_1, q_1) \neq d_n)).$$

(c) Construct a PTA which accepts the language $\overline{L(A)}$.