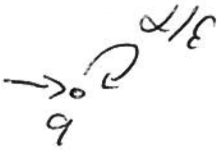
Examples on pre^* and accepting runs

Note that the pre^* computation only works correctly in the sense that

$$CF(A_{pre^*}) = pre^*(CF(A))$$

if the initial states of the P-NFA A do not have incoming arcs.

A flawed example

↳ Consider PDS $P =$ 

↳ Consider P-NFA $A =$ 

that represents the set of configurations $CF(A) = \{(q, \beta^n) \mid n \in \mathbb{N}\}$.

↳ The P-NFA does not obey the constraint on initial states.

↳ In fact, pre^* yields an incorrect result:

$$A_{pre^*} =$$ 

But

$$CF(A_{pre^*}) \not\subseteq pre^*(CF(A))$$

To check that the inclusion fails, consider

$$(q, \beta\alpha) \in CF(A_{pre^*}).$$

We have

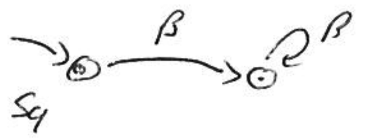
$$(q, \beta\alpha) \not\rightarrow$$

because no transition of P can remove β. But $(q, \beta\alpha)$ itself is not in $CF(A)$.

Correcting the flawed example:

↳ Unwind the loop in the P-NFA:

Represent $\{(q_1, \beta^k) \mid k \in \mathbb{N}\}$ by

$\mathcal{A} =$ 

↳ When we apply pre*, we obtain

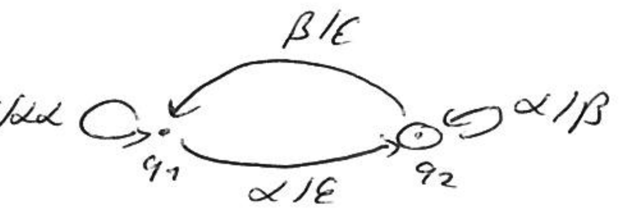
$\mathcal{A}_{pre^*} =$ 

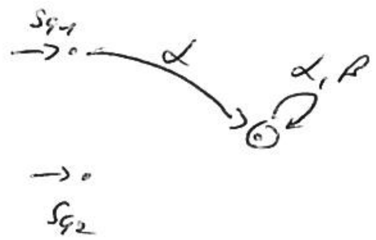
which is correct:

$$CF(\mathcal{A}_{pre^*}) = pre^*(CF(\mathcal{A})).$$

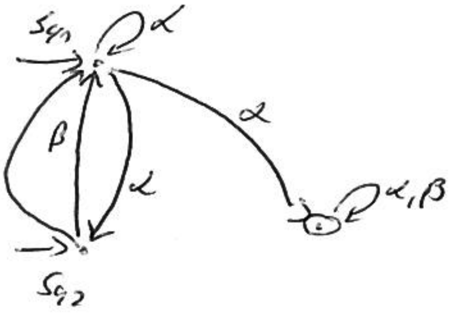
Example on accepting runs:

Consider

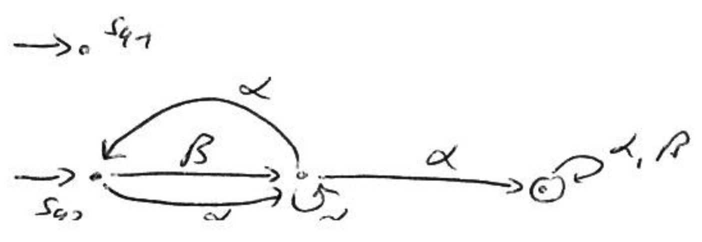
$BP =$ 

- Determine $pre^*(\{q_1\} \times \alpha\Gamma^*)$ where $\{q_1\} \times \alpha\Gamma^*$ is represented by $\mathcal{A} =$ 

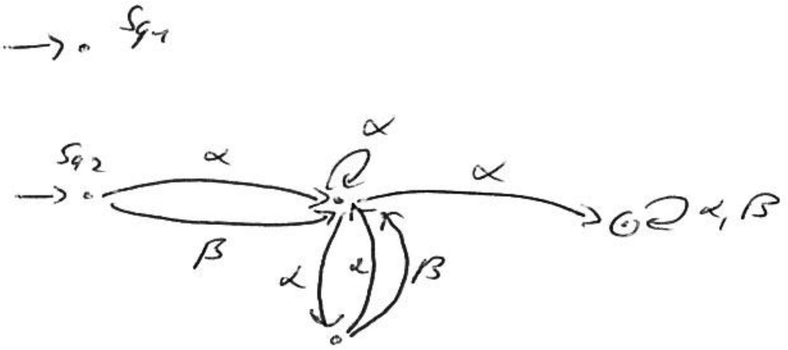
Then

$\mathcal{A}_{pre^*} =$ 

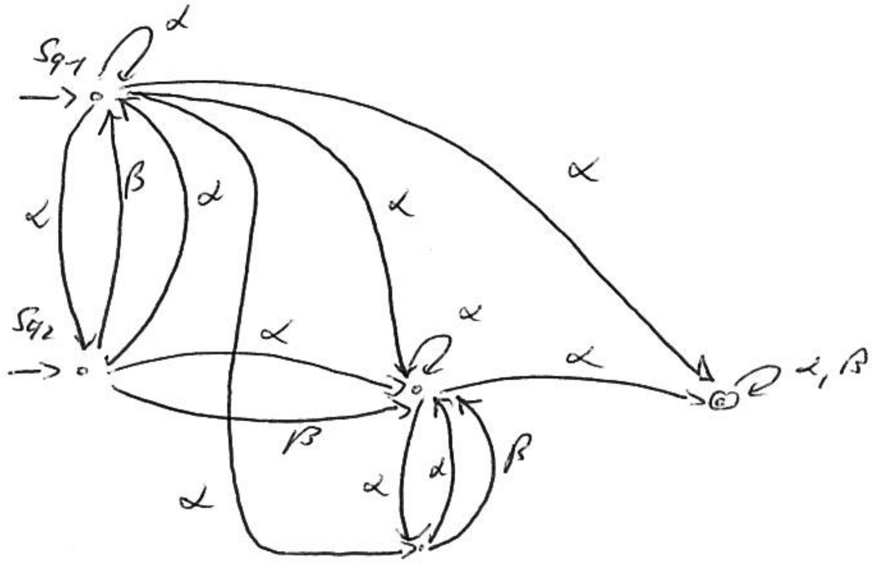
- Intersect $CF(\mathcal{A}_{pre^*}) \cap \{q_2\} \times \Gamma^* = CF(\mathcal{A}')$

where $\mathcal{A}' =$ 

- Use unwinding to construct a P-NFA that has no incoming arcs to initial states:



- Determine again pre*:



- Finally, compute another single pre: