Applied Automata Theory (WS 2014/2015) Technische Universität Kaiserslautern

Exercise Sheet 4

Jun.-Prof. Roland Meyer, Reiner Hüchting Due: Tue, Nov 25

Exercise 4.1 Presburger Formulas & Parikh Images

(a) Construct a finite automaton for the atomic Presburger formula $x - 3y \leq 1$.

(b) Present a Presburger formula ϕ such that every bound variable occurs in precisely one atomic expression and such that

$$Sol(\phi) = \left\{ \binom{2n+1}{n+3} \mid n \in \mathbb{N} \right\} \cup \left\{ \binom{3n+1}{2n+2} \mid n \in \mathbb{N} \right\}$$

(c) Give an NFA A so that $\Psi(L(A)) = Sol(\phi)$ for the Presburger formula ϕ from (b).

Exercise 4.2 Quantifier Elimination

Perform quantifier elimination for the following formula as described in the lecture notes:

$$\neg\forall x.\, 3x < 2y \vee y < 2x \ .$$

Exercise 4.3 Semilinear Sets

(a) Prove that semi-linear sets are Presburger definable: for any semi-linear set $S \subseteq \mathbb{N}^n$ there exists a Presburger formula φ_S such that $S = Sol(\varphi_S)$.

(b) A function $f\colon \mathbb{N}^n \to \mathbb{N}^m$ is linear if $f(x+y) = f(x)+f(y)$ and $f(kx) = kf(x)$ for all $k \in \mathbb{N}$. Prove that semi-linear sets are closed under linear functions, i.e. if $S \subseteq \mathbb{N}^n$ is semi-linear and $f\colon \mathbb{N}^n \to \mathbb{N}^m$ is a linear function then $f(S) \subseteq \mathbb{N}^m$ is semi-linear.

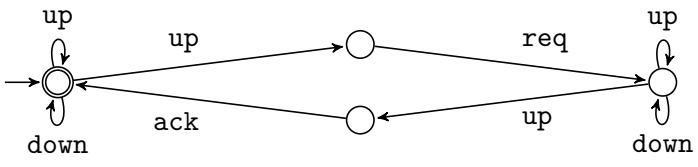
Exercise 4.4 Semilinear Extensions of Regular Languages

Consider extended regular expressions (r, S) where $r \in \mathrm{REG}_\Sigma$ and $S \subseteq \mathbb{N}^{|\Sigma|}$ is semilinear, and define $L(r, S) := \{w \in \Sigma^* \mid w \in L(r) \text{ and } \Psi(w) \in S\}$.

(a) Prove that emptiness of $L(r, S)$ is decidable for any extended regexp (r, S).

(b) Find an extended regexp (r, S) such that $a^n b^n c^n = L(r, S)$.

(c) What is the language r accepted by the "request/acknowledge" automaton below?



Describe the semilinear set S for which the extended regexp $L(r, S)$ represents $> 80\%$ system availability, i.e. the transition sequences with $\leq 20\%$ `down` time.