

Exercise Sheet 5

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Due: Tue, Dec 2

Exercise 5.1 Semilinear Sets

Let $S = \bigcup_{i \in \{1, \dots, l\}} \mathcal{L}(c_i, P_i) \subseteq \mathbb{N}^n$ be semi-linear. Prove closure under Kleene iteration:

$$\{v_1 + \dots + v_k \mid k \in \mathbb{N} \text{ and } v_1, \dots, v_k \in S\} = \bigcup_{I \subseteq \{1, \dots, l\}} \mathcal{L}\left(\sum_{i \in I} c_i, \bigcup_{i \in I} P_i \cup \{c_i\}\right).$$

Exercise 5.2 Parikh Images of Regular Languages

- (a) Prove that $\Psi(L)$ is semilinear if $L \in \text{REG}_\Sigma$.
- (b) Prove that for a semilinear set $S \subseteq \mathbb{N}^n$ there is a regular language L with $S = \Psi(L)$.

Exercise 5.3 Parikh Images of Context Free Languages

Use the method from class to compute $\Psi(L(G))$ for the grammar G whose rules are:

- (a) $S \rightarrow ab \mid S_1 S', S' \rightarrow S S_2, S_1 \rightarrow a, S_2 \rightarrow b$
- (b) $S \rightarrow S_1 S_2 \mid \varepsilon, S_1 \rightarrow a S b, S_2 \rightarrow b S c$