Exercise Sheet 9

Prof. Dr. Roland Meyer, Reiner Hüchting   Due: Tue, Jan 13

Exercise 9.1 LTL

(a) In the lecture notes, LTL was only defined with operators concerning the future. Now consider an operator $\lhd$ where $\lhd\, p$ means "p has held at some time in the past". Express the following formula without $\lhd$:

$$\Box\,(\varphi \to \lhd\, \psi)$$

(b) We define three notions of fairness (en and ex stand for "enabled" and "executed"):

$$
\begin{aligned}
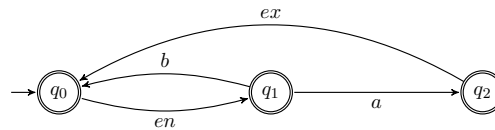\text{Absolute fairness (impartiality):} \quad & \Box\Diamond ex & (\textbf{AF}) \\
\text{Strong fairness (compassion):} \quad & \Box\Diamond en \to \Box\Diamond ex & (\textbf{SF}) \\
\text{Weak fairness (justice):} \quad & \Diamond\Box en \to \Box\Diamond ex & (\textbf{WF})
\end{aligned}
$$

Which of the following statements hold for the NBA A depicted below?

$$A \models \textbf{AF} \to \Box\Diamond a \qquad A \models \textbf{SF} \to \Box\Diamond a \qquad A \models \textbf{WF} \to \Box\Diamond a$$

Exercise 9.2 Unrollings

Prove the following equivalences:

$$\text{(a)} \quad \varphi\,\mathcal{U}\,\psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi\,\mathcal{U}\,\psi)) \qquad \text{(b)} \quad \varphi\,\mathcal{R}\,\psi \equiv \psi \wedge (\varphi \vee \bigcirc(\varphi\,\mathcal{R}\,\psi))$$

Exercise 9.3 Positive Normal Form

(a) Express $\neg\Big(\Box p \to \big((p \wedge \neg r)\,\mathcal{U}\, \neg \bigcirc q\big)\Big) \wedge \neg\big(\neg p \vee \bigcirc\Diamond r\big)$ in PNF.

(b) Prove that every LTL formula can be brought to PNF.

Exercise 9.4 GNBA-Construction

Construct GNBA for the following LTL-formulas:

(a) $\Theta_1 = p\,\mathcal{U}\,q$

(b) $\Theta_2 = (\neg p\,\mathcal{U}\,q) \vee \bigcirc(\neg q\,\mathcal{U}\,r)$