WS 2016/2017 25.10.2016

Exercises to the lecture
Complexity Theory
Sheet 1

Prof. Dr. Roland Meyer
Dr. Prakash Saivasan Delivery until 1.11.2016 at 10:00

Exercise 1.1 (*COPY* can be decided in quadratic time)

Let $\Sigma = \{a, b, \#\}$ be an alphabet. Recall the definition of the language *COPY* from the lecture:

$$COPY = \{w.\#.w \mid w \in \{a,b\}^*\}.$$

Show that *COPY* is in $\mathsf{DTIME}(n^2)$.

Hint: Construct a deterministic Turing Machine that decides COPY in quadratic time.

Exercise 1.2 (Crossing sequences of Turing Machines)

Let M be a Turing Machine and $x = x_1.x_2$, $y = y_1.y_2$ words over an alphabet Σ so that

$$CS(x, |x_1|) = CS(y, |y_1|).$$

Prove that $x_1.x_2 \in L(M)$ if and only if $x_1.y_2 \in L(M)$.

Exercise 1.3 (Θ, Ω and $\mathcal{O}$-Notation)

Let $g : \mathbb{N} \to \mathbb{N}$ be a function. Recall the following definitions from the exercise class:

$$\Theta(g(n)) = \left\{ f : \mathbb{N} \to \mathbb{N} \;\middle|\; \begin{array}{c} \text{there exist } c_1, c_2 > 0 \text{ and } n_0 \in \mathbb{N} \text{ such that} \\ 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \end{array} \right\},$$

$$\mathcal{O}(g(n)) = \left\{ f : \mathbb{N} \to \mathbb{N} \;\middle|\; \begin{array}{c} \text{there exist } c > 0 \text{ and } n_0 \in \mathbb{N} \text{ such that} \\ 0 \leq f(n) \leq c g(n) \text{ for all } n \geq n_0 \end{array} \right\},$$

$$\Omega(g(n)) = \left\{ f : \mathbb{N} \to \mathbb{N} \;\middle|\; \begin{array}{c} \text{there exist } c > 0 \text{ and } n_0 \in \mathbb{N} \text{ such that} \\ 0 \leq c g(n) \leq f(n) \text{ for all } n \geq n_0 \end{array} \right\}.$$

Show that the following equality of sets holds:

$$\Theta(g(n)) = \mathcal{O}(g(n)) \cap \Omega(g(n)).$$

Delivery until 1.11.2016 at 10:00 into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23