Exercises to the lecture
Complexity Theory
Sheet 6

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Dr. Prakash Saivasan Delivery until 13.12.2016 at 10h

Exercise 6.1 (Emptiness of context-free languages)

The **emptiness-problem for context-free languages** is the following problem:
Given: A context-free grammar G in Chomsky normal form.
Problem: Decide if $L(G)$ is empty.

a) Show that the emptiness-problem for context-free languages is in P.

b) Prove that the emptiness-problem is also P-hard with respect to logspace reductions. *Hint: Reduce CVP to (non-)emptiness of context-free languages.*

Exercise 6.2 (Safe Petri Nets)
Consider the following definitions:

- A **Petri Net** is a triple $N = (P, T, W)$, where P is a finite set of **places**, T is a finite set of **transitions** and $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a **weight function**.
- A **marking** of N is a map $M \in \mathbb{N}^{|P|}$ that maps places to natural numbers. Intuitively, a marking represents the number of *tokens* in all places.
- A transition t is **enabled** in a marking M if $M \geq W(-, t)$, where $W(-, t)$ denotes the vector $(W(p_1, t), \ldots, W(p_{|P|}, t))$. The vector $W(t, -)$ is defined similarly.
- If t is enabled in M, the transition can be **fired**: we obtain a new marking M' by subtracting $W(-, t)$ and adding $W(t, -)$. More formally, we write: $M \xrightarrow{t} M'$ if t is enabled in M and $M' = M - W(-, t) + W(t, -)$.
- If $\sigma = \sigma_1 \ldots \sigma_\ell$ is a sequence of transitions we also write $M \xrightarrow{\sigma} M'$ if there are markings $M_1, \ldots, M_{\ell+1}$ so that $M_1 = M$, $M_{\ell+1} = M'$ and $M_i \xrightarrow{\sigma_i} M_{i+1}$ for $i = 1, \ldots, \ell$.
- A marking M' is **reachable** from a marking M if there is a sequence of transitions σ so that $M \xrightarrow{\sigma} M'$.
- The Petri Net N is called **safe** from marking M if all markings reachable from M are in $\{0, 1\}^{|P|}$.

- The **reachability problem for safe Petri Nets** is defined as follows:
  Given: A Petri Net N, markings M, M' so that N is safe from M.
  Problem: Decide if M' is reachable from M.

The reachability problem for general Petri Nets is decidable but the only known decision procedure has *non-primitive recursive* complexity. For safe Petri Nets, we can do better:

a) Prove that the reachability problem for safe Petri Nets is in PSPACE.

b) Show that the problem is also PSPACE-hard with respect to polytime reductions.
   Hint: Don't try to reduce QBF to safe Petri Net reachability. Pick an arbitrary problem in PSPACE and transform its deterministic decider into a Petri Net.

Exercise 6.3 (Intersection-emptiness of regular languages)

The **intersection-emptiness problem for regular languages** is the following:
Given: NFAs $A_1, \ldots, A_k$ for some arbitrary $k \in \mathbb{N}$. Note that k is part of the input.
Problem: Decide if $\bigcap_{i=1}^{k} L(A_i)$ is empty.

a) Show that intersection-emptiness is in PSPACE.

b) Prove that intersection-emptiness is also PSPACE-hard with respect to polytime reductions.
   Hint: Reduce safe Petri Net reachability to intersection-emptiness. Note that an execution of a Petri Net $N = (P, T, W)$ is a sequence of firings. Firing a transition just amounts to putting and consuming tokens. Construct $|P| + 1$ automata over the alphabet $\{put_p, consume_p \mid p \text{ a place}\}$. For each $p \in P$ an automaton should check that the one token on p is used in the right way. The last automaton should mimic the behavior of the transitions.

Delivery until 13.12.2016 at 10h into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23