

Exercises to the lecture  
Complexity Theory  
Sheet 10

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Delivery until 24.01.2017 at 17h

**Exercise 10.1** (Parametrized *SAT*)

Consider the following parametrized version of *SAT*:

**Input:** A formula  $\varphi(x_1, \dots, x_k)$  of size  $n$ .

**Parameter:**  $k \in \mathbb{N}$ .

**Question:** Is there a satisfying assignment for  $\varphi$  ?

Construct a parametrized branching algorithm for the above problem and determine its runtime. Which part of the input makes *SAT* so expensive ?

**Exercise 10.2** (Unions of cliques)

A **clique** is a graph  $K = (V, E)$  such that for all  $u, v \in V$  we have:  $uv \in E$ . Hence, any pair of vertices has a connecting edge. The goal of this exercise is to show that the problem *CLUSTEREDITING* defined below is FPT.

**Input:** A graph  $G = (V, E)$ .

**Parameter:**  $k \in \mathbb{N}$ .

**Question:** Is it possible to add or delete at most  $k$  edges to turn the graph into a disjoint union of cliques ?

- a) Show that a graph  $G$  consists of disjoint cliques if and only if there are no three distinct vertices  $u, v, w \in V$  so that  $uv, vw \in E$  and  $uw \notin E$ .
- b) Prove that *CLUSTEREDITING* is FPT.  
*Hint: The criterion of Part a) can be used as a branching rule. So far, we have only considered binary branching trees. To solve the above problem, you may need a tree that has a bigger outdegree.*

**Delivery until 24.01.2017 at 17h into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23**