WS 2017/2018 24.10.2017

Exercises to the lecture
Complexity Theory
Sheet 2

Prof. Dr. Roland Meyer
M.Sc. Peter Chini Delivery until 02.11.2017 at 12h

Exercise 2.1 (Non-Emptiness of Context-Free Languages)

We consider the following problem:

Non-Emptiness of Context-Free Langauges (CFL Non − Empty)	
Input:	A context-free grammar G in Chomsky normal form.
Question:	Is $L(G)$ non-empty?

Show that CFL Non − Empty is P-complete with respect to logspace-many-one reductions.

Hint: You may reduce from CVP for the hardness.

Exercise 2.2

In this exercise, we want to show the NP-completeness of the following problem:

Triple Path Cover (TPC)	
Input:	A directed graph G.
Question:	Can we cover G with three disjoint paths?

More precise, TPC asks whether there are three paths

$$v_1^{(1)} \to v_2^{(1)} \to \cdots \to v_{n_1}^{(1)}$$
$$v_1^{(2)} \to v_2^{(2)} \to \cdots \to v_{n_2}^{(2)}$$
$$v_1^{(3)} \to v_2^{(3)} \to \cdots \to v_{n_3}^{(3)}$$

without repeating vertices such that each vertex v of G appears as a $v_j^{(i)}$ in exactly one of the paths.

Show that TPC is NP-complete. The hardness should be established by a reduction from the well-known NP-complete problem of finding a *Hamiltonian Cycle:*

Hamiltonian Cycle (Hamil Cycle)	
Input:	A directed graph G.
Question:	Is there a cycle in G (without repetition) that visits all vertices?

Exercise 2.3 (Safe Petri Nets)
Consider the following definition:

- A **Petri Net** is a triple $N = (P, T, W)$, where $P = \{p_1, \dots, p_{|P|}\}$ is a finite set of **places**, T is a finite set of **transitions** and $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a **weight function**.
- A **marking** of N is a map $M \in \mathbb{N}^{|P|}$ that maps places to natural numbers. Intuitively, a marking represents the number of *tokens* in all places.
- A transition t is **enabled** in a marking M if $M \geq W(-,t)$, where $W(-,t)$ denotes the vector $(W(p_1,t), \dots, W(p_{|P|},t))$. The vector $W(t,-)$ is defined similarly.
- If t is enabled in M, the transition can be **fired**: we obtain a new marking M' by subtracting $W(-,t)$ and adding $W(t,-)$. More formally, we write: $M \xrightarrow{t} M'$ if t is enabled in M and $M' = M - W(-,t) + W(t,-)$.
- If $\sigma = \sigma_1 \dots \sigma_\ell$ is a sequence of transitions we also write $M \xrightarrow{\sigma} M'$ if there are markings $M_1, \dots, M_{\ell+1}$ so that $M_1 = M$, $M_{\ell+1} = M'$ and $M_i \xrightarrow{\sigma_i} M_{i+1}$ for $i = 1, \dots, \ell$.
- A marking M' is **reachable** from a marking M if there is a sequence of transitions σ so that $M \xrightarrow{\sigma} M'$.
- The Petri Net N is called **safe** from marking M if all markings reachable from M are in $\{0,1\}^{|P|}$.

We are interested in the following problem.

Reachability for safe Petri Nets (Safe Reach)	
Input:	A Petri Net N, markings M, M' such that N is safe from M.
Question:	Is M' reachable from M?

Show that Safe Reach is PSPACE-complete.

Hint: Do not reduce QBF to Safe Reach. Pick an arbitrary problem in PSPACE, a problem decided by a polynomial-space-bounded TM and reduce it to Safe Reach. The cells of the TM's tape should then be simulated by places, the TM's transition relation gets simulated by the PN's transitions.

Delivery until 02.11.2017 at 12h into the box next to 343