Exercises to the lecture
Complexity Theory
Sheet 6

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Exercise 6.1 (Parameterized SAT)

Consider the following parameterized problem:

Boolean Satisfiability (SAT)	
Input:	A Boolean formula $\varphi(x_1, \dots, x_k)$.
Parameter:	$k \in \mathbb{N}$.
Question:	Is there a satisfying assignment for φ?

Construct a search tree for SAT and show that the problem is FPT.

Exercise 6.2 (Unions of cliques)

A *clique* is a graph $K = (V, E)$ such that for all $u, v \in V$ we have $uv \in E$. This means that any pair of vertices has a connecting edge. The following problem asks how *far away* a given graph is from being a union of cliques.

Cluster Editing (CLUSTER)	
Input:	A graph $G = (V, E)$ and a $k \in \mathbb{N}$.
Parameter:	$k \in \mathbb{N}$.
Question:	Is it possible to add or remove at most k edges to/from E such that the resulting graph is a disjoint union of cliques?

a) Show that a graph G consists of disjoint cliques if and only if there are no three distinct vertices $u, v, w \in V$ with $uv, vw \in E$ and $uw \notin E$.

b) Prove that CLUSTER is FPT.

Exercise 6.3 (Maximal satisfiability)

We construct a kernelization for the following problem:

Maximal Satisfiability (MAXSAT)	
Input:	A Boolean formula $\varphi = \bigwedge_{i=1}^{m} C_i$, where the C_i are clauses, and $k \in \mathbb{N}$.
Parameter:	$k \in \mathbb{N}$.
Question:	Is there a variable assignment that satisfies at least k clauses of φ?

Let (φ, k) be an instance of the problem. The first step of the kernelization is to delete all *trivial* clauses. We call a clause *trivial* if it contains a variable and its negation.

a) Show that by removing all trivial clauses, we can reduce (φ, k) to an instance (ψ, k') such that $k' \leq k$ and $(\psi, k') \in \mathsf{MAXSAT}$ if and only if $(\varphi, k) \in \mathsf{MAXSAT}$.

In the next step, we delete *long* clauses, clauses that contain more than k' literals.

b) Prove the following: If ψ contains more than k' long clauses, then $(\psi, k') \in \mathsf{MAXSAT}$.

c) Let t denote the number of long clauses in ψ and set $\hat{k} = k' - t$. Show that by removing all long clauses, we can reduce (ψ, k') to an instance $(\rho, \hat{k})$ such that $(\rho, \hat{k}) \in \mathsf{MAXSAT}$ if and only if $(\psi, k') \in \mathsf{MAXSAT}$.

Hence, we obtain an instance that only consists of clauses of size at most k'. We argue that we are only interested in such formulas the size of which is bounded by the parameter.

d) Prove the following: If $(\rho, \hat{k})$ has more than $2\hat{k}$ clauses, then $(\rho, \hat{k})$ is in MAXSAT.

e) Summarize the reduction steps in an algorithm and show that the size of the kernel (the size of the obtained instance) is bounded by $\mathcal{O}(k^2)$.

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