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Exercises to the lecture
Complexity Theory
Sheet 7

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Exercise 7.1 (Set Cover)

Consider the following problem:

Set Cover	
Input:	A family of sets $(S_i)_{i\in[1..m]}$ over a universe $U = \bigcup_{i\in[1..m]} S_i$ with n elements, and an $\ell \in \mathbb{N}$.
Parameter:	$\lvert U\rvert = n \in \mathbb{N}$.
Question:	Are there ℓ sets $S_{i_1}, \dots, S_{i_\ell}$ from the family such that $U = \bigcup_{j\in[1..\ell]} S_{i_j}$?

Develop an algorithm for Set Cover that relies on the Inclusion/Exclusion principle. Show that it runs in time $\mathcal{O}^*(2^n)$.

Hint: This is quite similar to the algorithm for computing the chromatic number.

Exercise 7.2 (Count TSP)

In this exercise, we want to establish an algorithm for the following problem:

Counting Traveling Salesperson (Count TSP)	
Input:	A complete (each two vertices are connected) graph $G = (V, E)$ and a weight function $w : E \to \{0, \dots, W\}$.
Parameter:	$\lvert V\rvert = n \in \mathbb{N}$.
Question:	What is the number of Hamiltonian cycles that admit minimal weight?

Let $\pi = v_0v_1 \dots v_k$ be a path or cycle in G. Then the weight of π is $w(\pi) = \sum_{i=0}^{k-1} w(v_iv_{i+1})$. The problem asks for the number of Hamiltonian cycles the weight of which is minimal among all Hamiltonian cycles.

Develop an algorithm for Count TSP based on the Inclusion/Exclusion principle, that runs in $\mathcal{O}^*(2^n)$ time.

Hint: It is easier if you fix the weight in the universe. For each weight j of a Hamiltonian cycle, define the universe U_j to be the cycles of length n, starting in v_0, of weight j. Then proceed as in the Inclusion/Exclusion-algorithm for Hamil Cycle. At some point, one has to count the number of cycles that have weight j in a certain graph. Use a dynamic programming approach for this task.

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