Exercises to the lecture
Complexity Theory
Sheet 7

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Exercise 7.1 (Emptiness of context-free languages)

The **emptiness-problem for context-free languages** is the following problem:
Given: A context-free grammar G in Chomsky normal form.
Problem: Decide if $L(G)$ is empty.

a) Show that the emptiness-problem for context-free languages is in P.

b) Prove that the emptiness-problem is also P-hard with respect to logspace reductions. *Hint: Reduce CVP to (non-)emptiness of context-free languages.*

Exercise 7.2 (Safe Petri Nets)
Consider the following definitions:

- A **Petri Net** is a triple $N = (P, T, W)$, where P is a finite set of **places**, T is a finite set of **transitions** and $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a **weight function**.
- A **marking** of N is a map $M \in \mathbb{N}^{|P|}$ that maps places to natural numbers. Intuitively, a marking represents the number of *tokens* in all places.
- A transition t is **enabled** in a marking M if $M \geq W(-, t)$, where $W(-, t)$ denotes the vector $(W(p_1, t), \ldots, W(p_{|P|}, t))$. The vector $W(t, -)$ is defined similarly.
- If t is enabled in M, the transition can be **fired**: we obtain a new marking M' by subtracting $W(-, t)$ and adding $W(t, -)$. More formally, we write: $M \xrightarrow{t} M'$ if t is enabled in M and $M' = M - W(-, t) + W(t, -)$.
- If $\sigma = \sigma_1 \ldots \sigma_\ell$ is a sequence of transitions we also write $M \xrightarrow{\sigma} M'$ if there are markings $M_1, \ldots, M_{\ell+1}$ so that $M_1 = M$, $M_{\ell+1} = M'$ and $M_i \xrightarrow{\sigma_i} M_{i+1}$ for $i = 1, \ldots, \ell$.
- A marking M' is **reachable** from a marking M if there is a sequence of transitions σ so that $M \xrightarrow{\sigma} M'$.
- The Petri Net N is called **safe** from marking M if all markings reachable from M are in $\{0, 1\}^{|P|}$.

- The **reachability problem for safe Petri Nets** is defined as follows:
  Given: A Petri Net N, markings M, M' so that N is safe from M.
  Problem: Decide if M' is reachable from M.

The reachability problem for general Petri Nets is decidable but the only known decision procedure has *non-primitive recursive* complexity. For safe Petri Nets, we can do better:

a) Prove that the reachability problem for safe Petri Nets is in PSPACE.

b) Show that the problem is also PSPACE-hard with respect to polytime reductions.
Hint: Don't try to reduce QBF to safe Petri Net reachability. Pick an arbitrary problem in PSPACE *and transform its deterministic decider into a Petri Net.*

Exercise 7.3 (Intersection-emptiness of regular languages)

The **intersection-emptiness problem for regular languages** is the following:
Given: NFAs $A_1, \ldots, A_k$ for some arbitrary $k \in \mathbb{N}$. Note that k is part of the input.
Problem: Decide if $\bigcap_{i=1}^{k} L(A_i)$ is empty.

a) Show that intersection-emptiness is in PSPACE.

b) Prove that intersection-emptiness is also PSPACE-hard with respect to polytime reductions.
Hint: Reduce safe Petri Net reachability to intersection-emptiness. Note that an execution of a Petri Net $N = (P, T, W)$ is a sequence of firings. Firing a transition just amounts to putting and consuming tokens. Construct $|P| + 1$ automata over the alphabet $\{put_p, consume_p \,|\, p \text{ a place}\}$. For each $p \in P$ an automaton should check that the one token on p is used in the right way. The last automaton should mimic the behavior of the transitions.

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