Exercises to the lecture
Complexity Theory
Sheet 11

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Exercise 11.1 (A circuit for finding satisfying assignments)

Assume we have a polynomial size circuit family $(C_n)_{n\in\mathbb{N}}$ that decides SAT. More precisely, $(C_n)_{n\in\mathbb{N}}$ solves the following problem:
Input: A formula $\varphi(x_0,\dots,x_k)$ encoded into input variables.
Note: the whole formula is the input of the circuit. The variables $x_0,\dots,x_k$ are only the variables of φ but these are not the input variables of the circuit.
Output: A variable s so that $s=1$ if and only if $\varphi(x_0,\dots,x_k)$ is in SAT.

Furthermore, assume that we have a polynomial size circuit family $(D_n)_{n\in\mathbb{N}}$ that is able to plug in values into a formula:
Input: A formula $\varphi(x_0,\dots,x_k)$ encoded into input variables and a variable v_0.
Output: The encoding for $\varphi(v_0,x_1\dots,x_k)$.

In the proof of Karp and Lipton's theorem we have seen the idea how to turn a circuit for SAT into a circuit that also finds a satisfying assignment for a given formula. Construct a polynomial size circuit family for this:
Input: A formula $\varphi(x_0,\dots,x_k)$ encoded into input variables.
Output: The variables s and $v_0,\dots,v_k$. So that:

- If $s=1$, then $\varphi(x_0,\dots,x_k)$ is in SAT and $v_0,\dots,v_k$ is a satisfying assignment.
- If $s=0$, then $\varphi(x_0,\dots,x_k)$ is not in SAT and $v_0=\cdots=v_k=0$.

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