Concurrency Theory (SS 2015) Out: Wed, May 27 Due: Tue, June 2

Exercise Sheet 6

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Problem 1: Petri Nets and wsts

(a) The transition system of a Petri net $N = (S, T, W, M_0)$ is $TS(N) := (R(N), M_0, \rightarrow)$. A transition $M_1 \rightarrow M_2$ exists if $M_1[t\rangle M_2$ for some $t \in T$. Show that $TS(N)$ is well-structured.

(b) Consider the following variant of Petri nets. A Petri net with zero-tests is a tuple $N = (S, T, W, Z, M_0)$ where S, T, W and M_0 are defined as in regular Petri nets, and $Z \subseteq (S \times T)$. A transition $t \in T$ is enabled in M if $M \geq W(—, t)$ and $M(s) = 0$ holds for each s such that $(s, t) \in Z$. The transition system of a Petri net with zero-tests $N = (S, T, W, Z, M_0)$ is $(R(N), M_0, \rightarrow)$ as above.

Argue whether the transition system of a Petri net with zero-tests is a wsts under the order $(\mathbb{N}^{|S|}, \leq)$.

Problem 2: Is any TS well-structured?

(a) Consider the set $\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\}$ and the order $\leq_\omega$ such that for all $n, n' \in \mathbb{N}$, $n \leq_\omega n'$ if $n \leq n'$, and for all $n \in \mathbb{N}_\omega$, $n \leq_\omega \omega$. Prove that $(\mathbb{N}_\omega, \leq_\omega)$ is a wqo.

(b) Take an arbitrary (finitely branching) transition system $TS = (\Gamma, \gamma_0, \rightarrow)$. Define $\ell(\gamma)$ for $\gamma \in \Gamma$ to be the length of the longest run $\gamma \rightarrow \gamma_1 \rightarrow \ldots$ in TS, or ω if there is an infinite run from γ. Prove that any $TS = (\Gamma, \gamma_0, \rightarrow)$ is well-structured under the order $\preccurlyeq$ where $\gamma \preccurlyeq \gamma'$ if $\ell(\gamma) \leq_\omega \ell(\gamma')$.

(c) Is $\preccurlyeq$ decidable in general?

Problem 3: Representing Upward/Downward-Closed Sets

Let $(A, \leq)$ be a wqo.

(a) Let $I \subseteq A$ be an upward closed set. Prove Lemma 6.2 given in class: if $Min(I)$ is the finite set of minimal elements of I, then $I = Min(I)\uparrow$.

(b) Consider the dual notion of *downward-closed set* D, i.e. for all $a \in A$, and $d \in D$, if $a \leq d$ then $a \in D$. Given $B \subseteq A$, we write $B\downarrow = \{a \in A \mid a \leq b, b \in B\}$. How can you finitely represent any downward-closed set $B \subseteq A$?
Hint: consider $A \setminus B$.

Problem 4: Termination for wsts

Given a wsts $(\Gamma, \rightarrow, \gamma_0, \leq)$, describe an algorithm to decide if every run from γ_0 is terminating or not. Assume the wsts to be finitely branching, i.e., for every configuration $\gamma_1 \in \Gamma$ there are finitely many $\gamma_2 \in \Gamma$ with $\gamma_1 \rightarrow \gamma_2$. Prove correctness of your algorithm.
Hint: start from the reachability tree for Petri nets and lift the construction for wsts.