Concurrency Theory (SS 2015) Out: Wed, 24 Jun Due: Tue, 30 Jun

Exercise Sheet 10

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Problem 1: Petri Net Languages

A labelled Petri net is a tuple $N = (S, T, W, M_0, X, \lambda, F)$ where S, T, W and M_0 are the finite set of places, transitions, the weight function and the initial marking respectively, defined as in ordinary Petri nets. The set X is a finite alphabet and the labelling function $\lambda\colon T \to X \cup \{\epsilon\}$ assigns a letter or the empty word to each transition. The set F is a finite set of final markings. We define the language generated by a labelled Petri net N to be the set

$$\mathcal{L}(N) = \{w \in X^* \mid \exists t_1, \ldots, t_n \in T :\ M_0 \,|t_1 \cdots t_n\rangle\, M_n \in F \wedge w = \lambda(t_1)\lambda(t_2)\cdots\lambda(t_n)\}.$$

Prove that the class of languages generated by labelled Petri nets is a full trio.

[Hint: *Show that it is closed under rational transductions.*]

Problem 2: Principal Trios

a) Let $\mathcal{C}$ and $\mathcal{D}$ be principal full trios. Show that the following holds: $\mathcal{C}$ and $\mathcal{D}$ are comparable ($\mathcal{C} \subseteq \mathcal{D}$ or $\mathcal{D} \subseteq \mathcal{C}$) if and only if $\mathcal{C} \cup \mathcal{D}$ is principal.

b) Let $\mathcal{C}_1 \subseteq \mathcal{C}_2 \subseteq \ldots$ be an infinite sequence of principal full trios. Show that $\bigcup_{i \in \mathbb{N}} \mathcal{C}_i$ is principal if and only if there is an $i \in \mathbb{N}$ such that $\mathcal{C}_j = \mathcal{C}_i$ for all $j \geq i$.

Problem 3: Shuffle vs Intersection

Given two languages L and K over X, we define their shuffle as

$$L \sqcup\!\sqcup K := \{u_0 v_0 \ldots u_n v_n \mid n \in \mathbb{N}, u_0, .., u_n, v_0, .., v_n \in X^*, u_0 \ldots u_n \in L, v_0 \ldots v_n \in K\}.$$

Let $\mathcal{C}$ be a full trio. Show that $\mathcal{C}$ is closed under $\sqcup\!\sqcup$ if and only if $\mathcal{C}$ is closed under $\cap$.

Problem 4: Complement vs Kleene

We define the complement of a language L as $\overline{L} = X^* \setminus L$, where X is the smallest alphabet such that $L \subseteq X^*$. Let $\mathcal{C}$ be a full trio. Show that if $\mathcal{C}$ is closed under complementation, then $\mathcal{C}$ is closed under Kleene iteration (L^*).

[Hint: *Try to construct* $\overline{(L\#)^*}$. *What does this language look like?*]

Problem 5: Regular Intersection

This is the extra problem, we will correct your submission and discuss it in the tutorial but you don't get a plus.

Let $\mathcal{C}$ be a language class that is closed under homomorphism (α), inverse homomorphism (α^{-1}) and concatenation with single letters from the left (L is transformed into cL) and from the right (L is transformed into Lc). Show that $\mathcal{C}$ is a full trio.

[Hint: *In order to simulate a finite automaton* $A = (Q, \Delta, q_0, \{q_f\})$ *over the alphabet* X *with edges* $\Delta \subseteq Q \times X \times Q$*, we can start by defining an homomorphism* $\alpha\colon \Delta^* \to X^*$*. From* $\alpha^{-1}(L)$ *try to construct the language of accepting runs encoded as words* $q_0 x_1 q_1 \cdots x_n q_n$.]