SS 2024 17.04.2024

Übungen zur Vorlesung
Concurrency Theory
Blatt 1

Prof. Dr. Roland Meyer
Jan Grünke

Abgabe bis 30.04.2024 um 23:59 Uhr

Aufgabe 1.1 (Marking Equation)

Let $N = (S, T, W)$ be a Petri net with connectivity matrix $\mathbb{C}$ and $M_1, M_2 \in \mathbb{N}^{|S|}$, $\sigma \in T^*$ such that $M_1[\sigma\rangle M_2$. Prove that $M_2 = M_1 + \mathbb{C} \cdot \psi(\sigma)$, where $\psi(\bullet)$ is the *Parikh image* function.
Hint: $\mathbb{C}(\bullet, t) = \mathbb{C} \cdot E_t$*, where* E_t *is the unit vector having* 1 *at position* t *and* 0 *elsewhere.*

Aufgabe 1.2 (1-safe Petri nets and Boolean programs)

Recall that a Petri net (N, M_0) is *1-safe* if we have $M \in \{0,1\}^P$ for all $M \in R(N, M_0)$. Consider *Boolean programs*, sequences of labeled commands over a fixed number of Boolean variables. For simplicity, we restrict ourselves to the following types of commands:

$z \leftarrow x \wedge y$	$z \leftarrow x \vee y$	$z \leftarrow \neg x$
`if` x `then goto` ℓ_t `else goto` ℓ_f	`goto` ℓ	`halt`

Here, x, y, z are variables and ℓ, ℓ_t, ℓ_f are labeles. The semantics of the commands are expected.
Assume that the initial variable assignment is given by $x = \mathit{false}$ for all variables x.
Assume that a Boolean program is given.
Explain how to construct an equivalent 1-safe Petri net. Equivalent means that the unique execution of the Boolean program is halting if and only if a certain marking is coverable.
Remark: This proves that coverability for 1-safe Petri nets is PSPACE-hard. In fact, coverability and reachability for 1-safe Petri nets are PSPACE-complete.

Aufgabe 1.3 (Petri nets, VASS, and VAS)

A *vector addition system with states (VASS)* of dimension $d \in \mathbb{N}$ is a tuple $A = (Q, \Delta, q_0, v_0)$ where Q is a finite set of control states, $\Delta \subseteq Q \times \mathbb{Z}^d \times Q$ is a set of transitions, $q_0 \in Q$ is the initial state and $v_0 \in \mathbb{N}^d$ is the initial counter assignment. We write transitions $(q, a, q') \in \Delta$ as $q \xrightarrow{a} q'$. A configuration of a VASS is a tuple (q, v) consisting of a control state $q \in Q$ and a counter assignment, a vector $v \in \mathbb{N}^d$. The initial configuration of interest is (q_0, v_0). A transition (q, a, q') is enabled in some configuration (q'', v) if $q'' = q$ and $(v + a) \in \mathbb{N}^d$ (i.e. $(v + a)_i \geq 0$ for all $i \in \{1, \ldots, d\}$). In this case, it can be fired, leading to the configuration $(q', v + a)$. Reachability is defined as expected.

(a) Let (N, M_0, M_f) be a Petri net. Show how to construct a VASS A and a configuration (q_f, v_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

(b) Let A be a VASS and (q_f, v_f) a configuration. Show how to construct a Petri net (N, M_0, M_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

(c) A *vector addition system (VAS)* is a VASS with a single state, i.e. $Q = \{q_0\}$. Show that VAS-reachability is interreducible with VASS reachability (or Petri net reachability).

Aufgabe 1.4 (Finiteness of the Coverability Graph)

In this excercise we prove finiteness of the coverability graph for a given Petri net. We do this in two steps:

(a) Prove *Dickson's Lemma*: For every $d \in \mathbb{N}$, $(\mathbb{N}^d, \leq_d)$ is *wqo*, where $(n_1, ..., n_d) \leq_d (m_1, ..., m_d)$ if for each $i = 1, ..., d$ we have $n_i \leq_i m_i$.

Hint: Show that for two wqo $(A, \leq_A)$, $(B, \leq_B)$ the product $(A \times B, \leq)$ with $(a, b) \leq (a', b') \iff a \leq_A a' \wedge b \leq_B b'$ is a wqo.

(b) Show that the search tree explored in the algorithm that constructs the coverability graph is finite. You can use the variant of Dickson's Lemma for generalized markings $\mathbb{N}^d_\omega$.

Abgabe bis 30.04.2024 um 23:59 Uhr per Mail an jan.gruenke1@tu-bs.de.