Concurrency Theory (WS 2010/11) Out: Thu, Jan 20 Due: Mon, Jan 24

Exercise Sheet 11

Jun.-Prof. Roland Meyer, Georgel Călin Technische Universität Kaiserslautern

Problem 1: Overapproximation Imitates WSTS

Let $TS = (\Gamma, \gamma_0, \rightarrow, \leq)$ be a wsts, (L, r) be an adequate domain of limits for $(\Gamma, \leq)$, and let $\Gamma' \subseteq \Gamma$ with $\gamma_0 \in \Gamma'$, respectively $L' \subseteq L$ with $\top \in L'$.

Consider a path $\gamma_0 \rightarrow \gamma_1 \rightarrow \ldots \rightarrow \gamma_k$ in TS and let $T = (N, n_r, \rightsquigarrow, \lambda)$ be an execution tree of $\texttt{Over}(TS, \Gamma', L')$.

Prove that there is a path $n_0 \rightsquigarrow n_1 \rightsquigarrow \ldots \rightsquigarrow n_{2k}$ with $n_0 = n_r$ in T such that $\gamma_i \in r(\lambda(n_{2i}))$ for any $i \in \{0, 1, \ldots, k\}$.

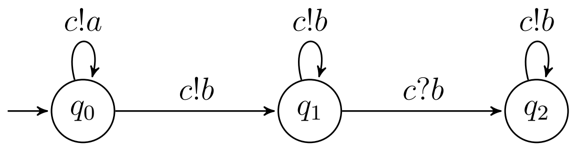
Problem 2: Adequate Domain of Limits for LCS

Let $\mathcal{L} = (Q, q_0, C, M, \rightarrow)$ be an lcs and $\leq$ the standard wqo on configurations.

Define an adequate domain of limits (*adl*) (L, r) for $(Q \times M^{*C}, \leq)$. Provide a method of enumerating the limits of the *adl* you suggested.

Problem 3: Expand, Enlarge, and Check

Consider the lcs depicted in the figure below:



Further consider the partial domains and limit sets described by

$$\Gamma_0 := \{(q_0, \epsilon), (q_0, a)\},\ \Gamma_1 := \Gamma_0 \cup \{(q_1, ab), (q_1, abb), (q_2, b)\}$$
$$L_0 := \{\top\} \cup \{(q_i, (a+b)^*) \mid i \in \{0, 1, 2\}\},\ L_1 := L_0 \cup \{(q_0, a^*), (q_1, a^*.b^*), (q_2, b^*)\}.$$

Iterate the EEC algorithm using the (Γ_0, L_0) and (Γ_1, L_1) above to determine reachability of:

- the upward-closed set $U_a = \{(q_2, a)\}\uparrow$.
- the upward-closed set $U_b = \{(q_2, b)\}\uparrow$.

In case one of the sets is unreachable, state the avoiding execution tree that proves it.

Problem 4: EEC for Something Different

Let $\Gamma = \{(i, j) \in \mathbb{N}^2 \mid i = j \vee j = i + 2\}$ and wsts $TS = (\Gamma, (0, 0), \rightarrow, \leq)$ with $\rightarrow$ defined by $(i, i) \rightarrow (i, i + 2)$ and $(i, i + 2) \rightarrow (i + 2, i + 2)$ for $i \in \mathbb{N}$, respectively with $\leq$ defined by $x \leq x + 2k \cdot (1, 1)$ for all $k \in \mathbb{N}$ and $x \in \Gamma \setminus \{(0, 0)\}$.

What is the coverability set of TS? Which are all the possible upward-closed sets in $(\Gamma, \leq)$? Find a limit set L' together with a partial domain Γ' of Γ which allows you to determine if

$$U_{2010} = \{x \in \Gamma \,|\, (2010, 2010) \leq x\} \text{ and } U_{2011} = \{x \in \Gamma \,|\, (2011, 2011) \leq x\}$$

are reachable or not using the EEC algorithm. What do the limits represent?