Concurrency Theory (WS 2010/11)	Out: Wed, Jan 26 Due: Mon, Jan 31

Exercise Sheet 12

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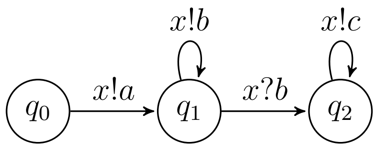
Problem 1: On Adequate Domains of Limits

For lcs $(Q, q_0, C, M, \rightarrow)$ take $L := \mathcal{P}(Q) \times sre(M)^C$ where $\mathcal{P}(\bullet)$ denotes the powerset.

(a) Why is $L \cap \Gamma = \emptyset$? *Hint: words versus atomic expressions.*
(b) Define the representation function r.
(c) Here, the $\top$ element can be expressed in the syntax. What is it?
(d) Show completeness of the domain of limits. Which theorem do you need?
(e) Describe how one can enumerate $L_0 \subseteq L_1 \subseteq L_2 \subseteq \ldots$

Problem 2: EEC and And-Or Graph Questions

Consider the lcs depicted in the figure below:



Further consider the partial domains and limit sets described by $\Gamma = \{(q_0, \epsilon), (q_1, a)\}$ and $L = \{\top\} \cup \{(q_1, (a+\epsilon)(b+c)^*), (q_1, (a+b)^*), (q_2, (b+c)^*), (q_2, (a+b)^*), (q_2, (a+b+c)^*\}$.
Construct the And-Or graph for the (Γ, L) overapproximation and:

(a) specify one of its execution trees that proves $\{(q_2, a)\}$ is avoidable.
(b) specify one of its execution trees that intersects $\{(q_2, a)\}$.

How many execution trees does the And-Or graph have?

Problem 3: π-calculus interpretation of FSA and PN

This exercise is meant to familiarize you with the behaviour (and expressiveness) of π-calculus.

Let $\mathcal{A} = (Q, q_0, \rightarrow)$ be an arbitrary finite state automaton. By using a free name q for each state $q \in Q$, a configuration (state at runtime) of $\mathcal{A}$ is represented by

$$\overline{q}\langle q\rangle \Big| \prod_{q \rightarrow q'} K_{q\rightarrow q'}\lfloor q, q'\rfloor,$$

where $K_{q\rightarrow q'}(q, q') := q(x).\left(K_{q\rightarrow q'}\lfloor q, q'\rfloor \,\middle|\, \overline{q'}\langle q'\rangle\right)$ describes the $q \rightarrow q'$ transition of $\mathcal{A}$.

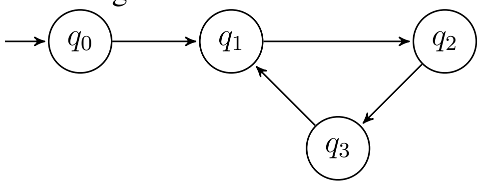
The automaton is then described by $\overline{q_0}\langle q_0\rangle \Big| \prod_{q\rightarrow q'} K_{q\rightarrow q'}\lfloor q, q'\rfloor$.

(a) Extend the described method to Petri nets. *Hints:*

−synchronize the execution of transitions; you need deadlocks

−change the defining equation $K_t(q, q') := \underline{\texttt{ here }} \left(K_t\lfloor q, q'\rfloor \middle| \underline{\texttt{ here }}\right)$

(b) Represent the *FSA* below using the method above:



What are your observations on the process syntax? What is the size of the processes as compared to the automaton (Petri net) they represent?

Problem 4: Structural Congruence & Normalization

(a) Show that $\nu a.P \equiv P$ if $a \notin fn(P)$. *Hint:* 0 *is useful.*

(b) Prove that the following two processes are structurally congruent:

$$P = \nu x \left(\nu s \left(\overline{x}\langle s\rangle.\overline{s}\langle a\rangle.\overline{s}\langle b\rangle \middle| x(u).u(y).u(z).\overline{y}\langle z\rangle\right) \mid x(t).t(w).t(v).\overline{v}\langle w\rangle\right)$$
$$Q = \nu x \left(\nu s \left(\overline{x}\langle s\rangle.\overline{s}\langle a\rangle.\overline{s}\langle b\rangle \middle| x(t).t(w).t(v).\overline{v}\langle w\rangle\right) \mid x(u).u(y).u(z).\overline{y}\langle z\rangle\right)$$

(c) Prove that each π-calculus process is structurally congruent to a process of the form

$$\nu x_1 \ldots \nu x_m. \left(P_1 | \ldots | P_n\right)$$

where each P_i is a choice.