Concurrency Theory (WS 2010/11)	Out: Wed, Nov 3 Due: Mon, Nov 8 (in class)

Exercise Sheet 2

Jun.-Prof. Roland Meyer, Georgel Călin
Technische Universität Kaiserslautern

Problem 1: Some Proofs

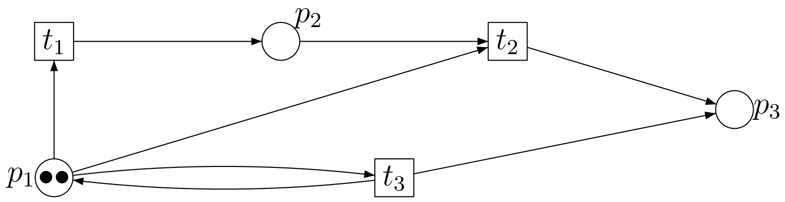
Let $N = (S, T, W, M_0)$ be a Petri net and $M, M_1, M_2 \in \mathbb{N}^S$ be markings of N.

(a) Prove that $R(N)$ is finite if and only if N is bounded.

(b) Prove that $\forall \sigma \in T^*$: if $M_1[\sigma\rangle M_2$ then $(M_1 + M)[\sigma\rangle(M_2 + M)$.

Problem 2: Boundedness & Termination: Decision Procedures

(a) Use the (**DFS**) algorithm given in class to decide if the following Petri net is bounded.



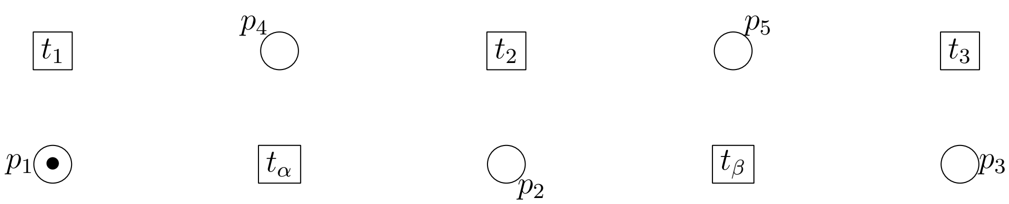
(b) Let $N = (S, T, W, M_0)$. Prove that:

- if there are $M_1, M_2 \in R(N)$ with $M_2 \geq M_1$ such that $M_0[\tau\rangle M_1[\sigma\rangle M_2$ for $\tau \in T^*$ and $\sigma \in T^+$ then N does not terminate.
- if N does not terminate then there are $M_1, M_2 \in R(N)$ with $M_2 \geq M_1$ such that $M_0[\tau\rangle M_1[\sigma\rangle M_2$ for $\tau \in T^*$ and $\sigma \in T^+$.

With these results in mind, devise an algorithm for deciding termination.

Problem 3: Firing Sequences and Petri Net Languages

Consider the Petri net $N = (S, T, W, M_0)$ below, where the arcs are intentionally left out.



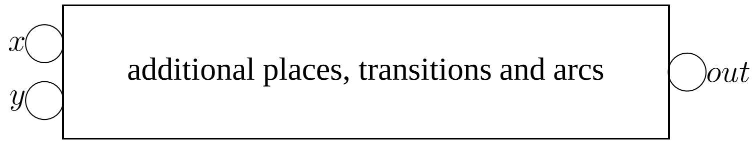
Add arcs to N such that M with $M(p_3) = 1$ (and $M(p) = 0$ if $p \neq p_3$) is reachable and:

(a) $\{\sigma \in T^* \mid M_0[\sigma\rangle M\} = \{t_1^m t_\alpha t_2^n t_\beta t_3^{m+n} \mid m, n \in \mathbb{N}\}$;
(b) $\{\sigma \in T^* \mid M_0[\sigma\rangle M\} = \{t_1^{m+n} t_\alpha t_2^m t_\beta t_3^n \mid m, n \in \mathbb{N}\}$;
(c) $\{\sigma \in T^* \mid M_0[\sigma\rangle M\} = \{t_1^n t_\alpha t_2^n t_\beta t_3^n \mid n \in \mathbb{N}\}$.

How does this relate to context-free languages?

Problem 4: Addition and Multiplication as Petri Nets

Consider the Petri net which contains places x, y and out as in the picture below.



Add places and transitions to your liking such that:

(a) if $M_0(x) = m$, $M_0(y) = n$ and $M_0(out) = 0$ the Petri net always terminates with marking M_{ter} such that $M_{ter}(out) = m + n$.

(b) if $M_0(x) = m$, $M_0(y) = n$ and $M_0(out) = 0$ the Petri net always terminates with marking M_{ter} such that $M_{ter}(out)$ is any of $\{0, \ldots, m \cdot n\}$.

In both cases provide the initial marking of the additional places you might have added and argument why your Petri nets do what they should.