Concurrency Theory (WS 2010/11) Out: Wed, Nov 10 Due: Mon, Nov 15

Exercise Sheet 3

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Problem 1: Marking Equation

Let $N = (S, T, \mathbb{F}, \mathbb{B})$ be a Petri net with connectivity matrix $\mathbb{C}$ and $M_1, M_2 \in \mathbb{N}^{\|S\|}$, $\sigma \in T^*$ such that $M_1[\sigma\rangle M_2$. Prove that $M_2 = M_1 + \mathbb{C} \cdot p(\sigma)$, where $p(\bullet)$ is the *Parikh image* function.

Hint: $\mathbb{C}(\bullet, t) = \mathbb{C} \cdot E_t$*, where* E_t *is the unit vector having* 1 *at position* t *and* 0 *elsewhere.*

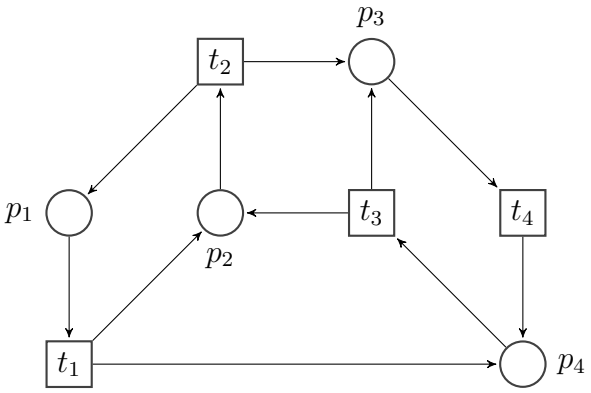
Problem 2: S/T-Invariants for Petri Nets

Let $N = (S, T, W)$ be a Petri net.

(a) prove that if I and J are structural (S-) invariants of N, so are $I + J$ and $k \cdot I$ ($\forall k \in \mathbb{Z}$).

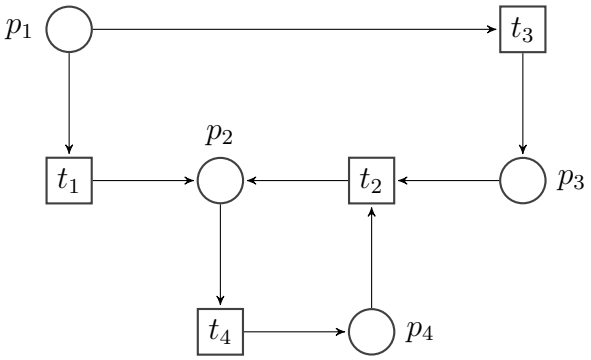
(b) prove that if I and J are transition (T-) invariants of N, so are $I + J$ and $k \cdot I$ ($\forall k \in \mathbb{N}$).

(c) compute a basis of S-invariants for the following net:



Problem 3: Traps and Siphons

Determine the traps and siphons in the following net:



Compute the net's trap matrix $\mathbb{C}_Q$ and verify your findings against the trap/siphon equation.

Problem 4: Family of Generating Traps

Add arcs to the Petri net N below so that its family of generating traps contains exponentially (in N's size) many traps. Once added, describe $N = (S, T, W)$ formally and prove that the family of generating traps is exponential in N's size.

Does the generating family of traps for N contain only minimal traps? Argument your answer.