Concurrency Theory (WS 2010/11) Out: Wed, Dec 8 Due: Mon, Dec 13

Exercise Sheet 7

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Problem 1: $\mathbb{N}^k$ is WQO & Petri Nets as WSTS

(a) Prove that $(\mathbb{N}^k, \leq)$ is a wqo for all $k \in \mathbb{N}$. Note that $(\mathbb{N}, \leq)$ occurs as base case.

(b) The transition system of a Petri net $N = (S, T, W, M_0)$ is $TS(N) := (R(N), \rightarrow, M_0)$. A transition $M_1 \rightarrow M_2$ exists if $M_1[t\rangle M_2$ for some $t \in T$. Show that $TS(N)$ is well-structured.

Problem 2: Lossy Channel Systems, WQO, and WSTS

Consider some lcs $L = \langle Q, q_0, C, M, \rightarrow\rangle$. Prove that

(a) $(Q \times M^{*C}, \leq)$ with $\leq$ as defined in the lecture is a wqo

(b) $(TS(L), \leq)$ is a wsts.

Problem 3: Parallel Composition of WSTS

Consider two wsts $TS_1 = (\Gamma_1, \rightarrow_1, \gamma_0, \leq_1)$ and $TS_2 = (\Gamma_2, \rightarrow_2, \overline{\gamma_0}, \leq_2)$. Define their parallel composition to be $TS_1 \parallel TS_2 := (\Gamma_1 \times \Gamma_2, \rightarrow, \gamma_0 \times \overline{\gamma_0})$ where

$$(\gamma_1, \overline{\gamma_1}) \rightarrow (\gamma_2, \overline{\gamma_2}) \text{ if } \gamma_1 \rightarrow_1 \gamma_2 \text{ and } \overline{\gamma_1} \rightarrow_2 \overline{\gamma_2}.$$

Prove that $(TS_1 \parallel TS_2, \leq_1\leq_2)$ is a wsts.

Problem 4: Termination for WSTS

Lift the decision procedure for termination of Petri nets (Exercise Sheet 2, Problem 2) to wsts. You have to assume the wsts $(\Gamma, \rightarrow, \gamma_0, \leq)$ to be finitely branching, i.e., for every configuration $\gamma_1 \in \Gamma$ there are finitely many $\gamma_2 \in \Gamma$ with $\gamma_1 \rightarrow \gamma_2$. Prove correctness of your algorithm.