$$
\begin{array}{rcl}
ENV(cfa) & := & FA\lfloor cfa\rfloor \mid ENV\lfloor cfa\rfloor \\
MRG(cfa) & := & cfa(id_x).id_x(ca_x).id_x(rq_x). \\
 & & \qquad cfa(id_y).id_y(ca_y).id_y(rq_y).\overline{ca_y}\langle rq_x\rangle.MRG\lfloor cfa\rfloor \\
FA(cfa) & := & \nu id, ca, rq.\overline{cfa}\langle id\rangle.\overline{id}\langle ca\rangle.\overline{id}\langle rq\rangle. \\
 & & \qquad (\ ca(rqnl).RQ\lfloor id, rqnl\rfloor + rq(nf).\overline{nf}\langle id\rangle.LD\lfloor id, nf\rfloor\ ) \\
RQ(id, rqnl) & := & \overline{rqnl}\langle id\rangle.id(nl).FL\lfloor id, nl\rfloor.
\end{array}
$$

Table 6.1: π-Calculus model of the merge manoeuvre.

The structural semantics of $ENV\lfloor cfa\rfloor \mid MRG\lfloor cfa\rfloor$ is depicted in Figure 6.2. We explain the meanings of places and transitions. Initially, the processes $F_1 = ENV\lfloor cfa\rfloor$ and $F_4 = MRG\lfloor cfa\rfloor$ are present and so the corresponding places are marked. With transition t_1, the environment process $ENV\lfloor cfa\rfloor$ generates free agents $F_2 = FA\lfloor cfa\rfloor$. Transition t_2 represents a call to the process identifier FA, which yields

$$F_3 \quad = \quad \nu id, ca, rq.\overline{cfa}\langle id\rangle.\overline{id}\langle ca\rangle.\overline{id}\langle rq\rangle.choice.$$

We use *choice* as a shortcut, which is replaced by the choice composition

$$ca(rqnl).RQ\lfloor id, rqnl\rfloor + rq(nf).\overline{nf}\langle id\rangle.LD\lfloor id, nf\rfloor$$

to obtain the full definition of F_3. Shortcuts improve the readability of processes, they are not part of the π-Calculus syntax. The call $MRG\lfloor cfa\rfloor$, represented by transition t_3, gives

$$F_5 \quad = \quad cfa(id_x).id_x(ca_x).id_x(rq_x).reg_y.\overline{ca_y}\langle rq_x\rangle.MRG\lfloor cfa\rfloor,$$

where reg_y abbreviates

$$cfa(id_y).id_y(ca_y).id_y(rq_y).$$

With t_4 the first free agent passes its id to the MRG process. The transition consumes a token for a free agent F_3, and the token for the MRG process F_5, and produces a fragment that contains a free agent and the MRG process, F_6. With transitions t_5 and t_6, the free agent continues to pass its ca and rq channels, resulting in fragments F_7 and F_8:

$$
\begin{array}{rcl}
F_6 & = & \nu id.\big(\nu ca, rq.\overline{id}\langle ca\rangle.\overline{id}\langle rq\rangle.choice \\
 & & \qquad \mid id(ca_x).id(rq_x).reg_y.\overline{ca_y}\langle rq_x\rangle.MRG\lfloor cfa\rfloor\big)
\end{array}
$$

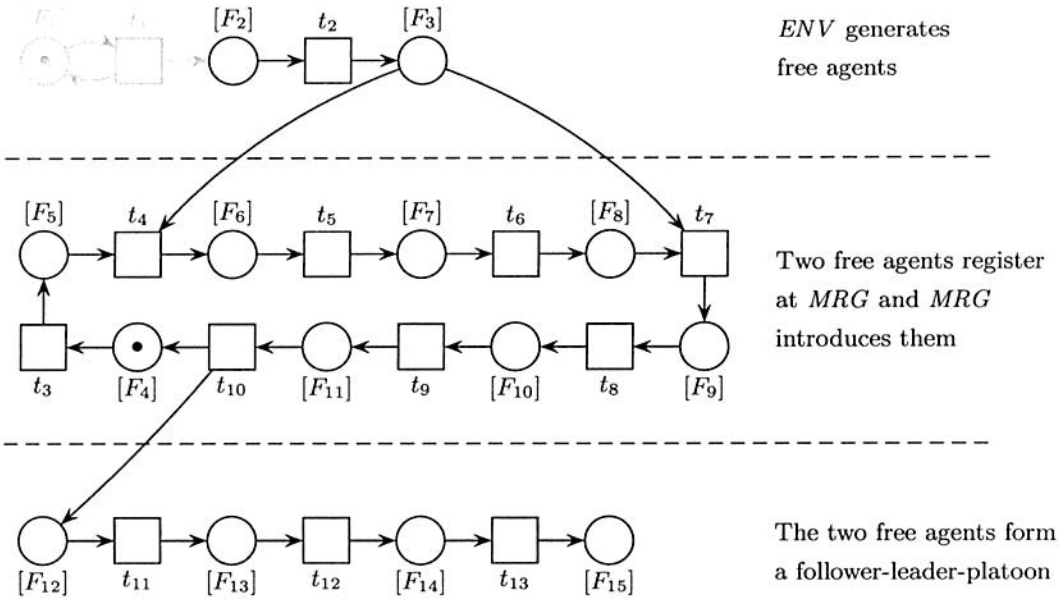

Figure 6.2:
Structural semantics $\mathcal{N}[\![ENV \lfloor cfa \rfloor \mid MRG \lfloor cfa \rfloor]\!]$ of the merge manoeuvre. The fragment definitions are given in the text. Cutting away the gray part yields a subnet where the number of cars stays constant.

$$F_7 = \nu id.\big(\nu ca, rq.\overline{id}\langle rq\rangle.choice \mid id(rq_x).reg_y.\overline{ca_y}\langle rq_x\rangle.MRG\lfloor cfa\rfloor\big)$$
$$F_8 = \nu rq.\big(\nu id, ca.choice \mid reg_y.\overline{ca_y}\langle rq\rangle.MRG\lfloor cfa\rfloor\big).$$

The registration of the second free agent, which yields F_9, F_{10}, and F_{11} given below, is similar. We use α-conversion to rename the id, ca, and rq channels of the first free agent to id_1, ca_1, and rq_1. The shortcuts $choice_1$ and $choice_2$ correspond to $choice$ with those names changed accordingly:

$$\begin{aligned}
F_9 &= \nu id_2.\big(\ \nu rq_1.(\nu id_1, ca_1.choice_1 \mid id_2(ca_y).id_2(rq_y).\overline{ca_y}\langle rq_1\rangle.MRG\lfloor cfa\rfloor) \\
&\qquad \mid \nu ca_2, rq_2.\overline{id_2}\langle ca_2\rangle.\overline{id_2}\langle rq_2\rangle.choice_2\ \big) \\
F_{10} &= \nu ca_2, id_2.\big(\ \nu rq_1.(\nu id_1, ca_1.choice_1 \mid id_2(rq_y).\overline{ca_2}\langle rq_1\rangle.MRG\lfloor cfa\rfloor) \\
&\qquad \mid \nu rq_2.\overline{id_2}\langle rq_2\rangle.choice_2\ \big) \\
F_{11} &= \nu ca_2.\big(\ \nu rq_1.(\nu id_1, ca_1.choice_1 \mid \overline{ca_2}\langle rq_1\rangle.MRG\lfloor cfa\rfloor) \\
&\qquad \mid \nu id_2, rq_2.(ca_2(rqnl).RQ\lfloor id_2, rqnl\rfloor + \ldots)\ \big).
\end{aligned}$$

In F_{11}, the MRG process is ready to pass the rq_1 channel of the leading car to the second free agent. Since it uses the ca_2 channel, we say that the MRG process *sends a car ahead message*. Afterwards, the process $MRG\lfloor cfa\rfloor$ forgets the restricted names of both free agents and the fragment is split up. The free

agent that receives the car ahead message becomes an $RQ\lfloor id_2, rq_1 \rfloor$ process in F_{12}. With transition t_{11} from F_{12} to F_{13}, RQ is replaced by its defining process:

$$\begin{aligned} F_{12} &= \nu rq_1.\big(\nu id_1, ca_1.choice_1 \mid \nu id_2.RQ\lfloor id_2, rq_1 \rfloor\big) \\ F_{13} &= \nu rq_1.\big(\nu id_1, ca_1.(\ldots + rq_1(nf).\overline{nf}\langle id_1\rangle.LD\lfloor id_1, nf \rfloor) \\ &\qquad \mid \nu id_2.\overline{rq_1}\langle id_2\rangle.id_2(nl).FL\lfloor id_2, nl \rfloor \big). \end{aligned}$$

In F_{13}, the second free agent issues a request to merge with the leading car. Transition t_{12} from F_{13} to F_{14} models the acceptance of this request as it reflects the communication of both cars on the rq_1 channel. With t_{13}, the now leader passes its id_1 channel to the follower, which yields the car platoon in F_{15}.

$$\begin{aligned} F_{14} &= \nu id_2.\big(\nu id_1.\overline{id_2}\langle id_1\rangle.LD\lfloor id_1, id_2 \rfloor \mid id_2(nl).FL\lfloor id_2, nl \rfloor\big) \\ F_{15} &= \nu id_1, id_2.\big(LD\lfloor id_1, id_2 \rfloor \mid FL\lfloor id_2, id_1 \rfloor\big). \end{aligned}$$

We continue with the investigation of the occurrence numbers of processes in the car platoon system.