Concurrency Theory (WS 2011/12)	Out: Tue, Dec 6 Due: Mon, Dec 12

Exercise Sheet 8

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Problem 1: Petri Nets and LCSs as WSTS

(a) The transition system of Petri net $N = (S, T, W, M_0)$ is $TS(N) := (\mathbb{N}^{|S|}, M_0, \rightarrow)$, where transition $M_1 \rightarrow M_2$ exists if $M_1[t\rangle M_2$ for some $t \in T$. Prove that $(\mathbb{N}^k, \leq)$ is a wqo for any $k \in \mathbb{N}$ and that $TS(N)$ is well-structured for any net N.

(b) Consider some lcs $L = \langle Q, q_0, C, M, \rightarrow \rangle$. Prove that $(Q \times {M^*}^C, \leq)$, with $\leq$ as defined in the lecture, is a wqo and $(TS(L), \leq)$, with $TS(L) := (Q \times {M^*}^C, \gamma_0, \rightarrow)$, is well structured.

Problem 2: Upward-Closed Sets by Minimal Elements

Let $(A, \leq)$ be a wqo and let $I \subseteq A$ be an upward closed set. Prove Lemma 6.2 given in class: if $Min(I)$ is a finite set of minimal elements of I, then $I = Min(I)\uparrow$.

Problem 3: Parallel Composition of WSTS

Consider two wsts $TS_1 = (\Gamma_1, \gamma_0, \rightarrow_1, \leq_1)$ and $TS_2 = (\Gamma_2, \overline{\gamma_0}, \rightarrow_2, \leq_2)$. Define their parallel composition to be $TS_1 \parallel TS_2 := (\Gamma_1 \times \Gamma_2, (\gamma_0, \overline{\gamma_0}), \rightarrow)$ where

$$(\gamma_1, \overline{\gamma_1}) \rightarrow (\gamma_2, \overline{\gamma_2}) \text{ if } \gamma_1 \rightarrow_1 \gamma_2 \text{ and } \overline{\gamma_1} \rightarrow_2 \overline{\gamma_2}.$$

Prove that $(TS_1 \parallel TS_2, \leq_{1\times 2})$ is a wsts.

Problem 4: LCS Variation remains Well Structured

Consider another type of lcs $L = (Q, q_0, \{c\}, M, \rightarrow)$ with c a channel carrying natural numbers as content, i.e., $M = \mathbb{N}$. Take the ordering $\leq^* \subseteq M^* \times M^*$ given in Higman's lemma.

(a) Prove that $(Q \times M^*, \lhd)$, with $\lhd$ defined by $(q, w) \lhd (q, w')$ iff $w \leq^* w'$, is a wqo.

(b) The transitions in L are given by $q \xrightarrow{!n} q'$ and $q \xrightarrow{?n} q'$ with $n \in \mathbb{N}$. The first appends n to the channel, the second receives a number $n' \geq n$ with $n' \in \mathbb{N}$ from the head of the channel. The channel is supposed to be lossy. Formalise the transition relation between configurations.

(c) Prove that $((Q \times M^*, (q_0, \epsilon), \rightarrow), \lhd)$ is a wsts.