

Exercise Sheet 11

Problem 1: Reachability of Upward-Closed Sets

Let $(\Gamma, \gamma_0, \rightarrow, \leq)$ be a well-structured transition system and $I \subseteq \Gamma$ an upward-closed set.

- (a) Prove that $R(\gamma_0) \cap I = \emptyset$ if and only if $R(\gamma_0)\downarrow \cap I = \emptyset$.
- (b) Let $R(\gamma_0) \cap I \neq \emptyset$ in $(\Gamma, \gamma_0, \rightarrow)$. Prove that there exists $\Gamma' \subseteq \Gamma$ finite with $\gamma_0 \in \Gamma'$ such that $R(\gamma_0) \cap I \neq \emptyset$ in $(\Gamma', \gamma_0, \rightarrow \cap (\Gamma' \times \Gamma'))$.

Problem 2: Adequate Domain of Limits for LCSs

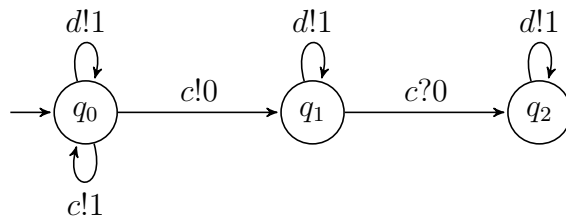
- (a) Show that symbolic configurations (q, R) with $R: C \rightarrow SRE$ are an adl.
- (b) Argue why the above adequate domain of limits (adl) is effective for LCSs.

Problem 3: And-Or Graphs and Execution Trees

Give four And-Or graphs with the following properties: the first one has infinitely many execution trees, the second one has more than one but finitely many execution trees, the third has a unique execution tree with infinitely many branches, and the last has a unique execution tree with finitely many branches.

Problem 4: Expand, Enlarge and Check

Consider the lossy channel system LCS :



together with $\Gamma = \{(q_0, \varepsilon), (q_1, \varepsilon), (q_2, \varepsilon)\}$ and limit domains

$$L_0 = \left\{ \top, \left(q_0, \begin{pmatrix} 1^* \\ \varepsilon \end{pmatrix} \right), \left(q_0, \begin{pmatrix} \varepsilon \\ 1^* \end{pmatrix} \right), \left(q_1, \begin{pmatrix} (0+1)^* \\ 0^*.1^* \end{pmatrix} \right), \left(q_1, \begin{pmatrix} (0+1)^* \\ 1^*.0^* \end{pmatrix} \right) \right\}$$

$$L_1 = L_0 \cup \left\{ \left(q_0, \begin{pmatrix} 1^* \\ 1^* \end{pmatrix} \right), \left(q_1, \begin{pmatrix} 1^*. (0+\varepsilon) \\ 1^* \end{pmatrix} \right), \left(q_2, \begin{pmatrix} \varepsilon \\ 1^* \end{pmatrix} \right) \right\}.$$

- (a) Compute $Over(LCS, \Gamma, L_0)$. Provide an execution tree.
- (b) Compute $Over(LCS, \Gamma, L_1)$. Argue why configuration $\left(q_2, \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} \right)$ is not coverable.