Concurrency Theory (WS 2016) Out: Thu, 17 Nov Due: Wed, 23 Nov

Exercise Sheet 4

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Problem 1: Decision Procedure for Place Boundedness

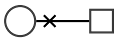
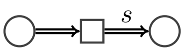
Consider a Petri net $N = (S, T, W, M_0)$ and let $Cov(N) = (V, E, M_0)$ be its coverability graph. Prove that $s \in S$ is unbounded if and only if there exists $L \in V$ with $L(s) = \omega$.

Petri net extensions Let A be the set of actions $A_S := \{(\mathbf{p}, n, m) \mid n, m \in \mathbb{N}\} \cup \{(\mathbf{t}, s) \mid s \in S\} \cup \{\mathbf{r}\}$. An extended Petri net is a tuple (S, T, φ), where S is the set of places, T is the set of transitions and $\varphi\colon T \times S \to A_S$. We define the relation $M_1 \xrightarrow{t} M_2$ to hold when for all $s \in S$, if $\varphi(t, s) = (\mathbf{p}, x, y)$ then $M_1(s) \geq x$, and

$$M_2(s) = \begin{cases} 0 & \text{if } \varphi(t,s) = \mathbf{r} \\ M_1(s) + M_1(s') & \text{if } \varphi(t,s) = (\mathbf{t}, s') \\ M_1(s) - x + y & \text{if } \varphi(t,s) = (\mathbf{p}, x, y) \end{cases}$$

A *reset net* is an extended Petri net where $\varphi(t, s) \neq (\mathbf{t}, \_)$, for all $t \in T, s \in S$.
A *transfer net* is one where, for all $t \in T$ and $s' \in S$, $\varphi(t, s') = \mathbf{r}$ if and only if there is a $s \in S$ with $\varphi(t, s) = (\mathbf{t}, s')$.

The intuition is that firing a transition t with $\varphi(t, s) = \mathbf{r}$ resets the number of tokens in s, while $\varphi(t, s) = (\mathbf{t}, s')$ and $\varphi(t, s') = \mathbf{r}$ makes t transfer *all* the tokens from s to s'; $\varphi(t, s) = (\mathbf{p}, x, y)$ is a standard Petri net arc with $W(s, t) = x$ and $W(t, s) = y$.

Graphically, a reset arc is represented with a cross: s  and transfer arcs with double lines: s  s' .

Problem 2: Termination

We say that *monotonicity* holds for a class of extended Petri nets if for all markings M_1, M_2, M_1', if $M_1 \xrightarrow{\sigma} M_2$ and $M_1' \geq M_1$ then there is a marking $M_2' \geq M_2$ such that $M_1' \xrightarrow{\sigma} M_2'$.

a) Prove monotonicity holds for transfer nets

b) Prove monotonicity holds for reset nets

c) Does this allow us to decide termination of transfer/reset nets as in Petri nets? Justify.

Problem 3: Boundedness

Similarly, *strict monotonicity* holds if for all markings M_1, M_2, M_1', if $M_1 \xrightarrow{\sigma} M_2$ and $M_1' > M_1$ then there is a marking $M_2' > M_2$ such that $M_1' \xrightarrow{\sigma} M_2'$.

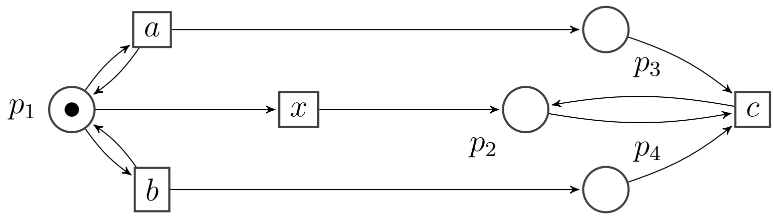
a) Prove strict monotonicity holds for transfer nets,

b) Does this imply decidability of boundedness for transfer nets? Justify.

c) Give a counterexample for the same property in the case of reset nets.

Problem 4: Rackoff

Given the marked Petri net below and a target marking $T = (1, 0, 10, 100)$, calculate the values of $m(3, (1, 0, 0, 0))$ as well as $f(3)$ and argue why they are correct.



Since the Petri net above is not in "VAS form" (i.e. with ${}^\bullet t \cap t^\bullet = \emptyset$), you can either transform it so it is, or simply check that the shortest paths you consider are all valid firing sequences.

Problem 5: Upper bound for boundedness (Optional)

Argue whether Rackoff's result can be used to derive an upper bound for deciding boundedness of usual Petri nets.

[Hint: *review the algorithm for boundedness*]