# Concurrency theory
# Exercise sheet 9

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Winter term 2017/18

Out: December 21 | Due: January 9

Submit your solutions until Tuesday, January 9, during the lecture.

**Exercise 1: Sequential consistency**

In the memory model **SC (sequential consistency)**, we assume that access to the main memory is atomic. More formally, the transition relation $\rightarrow_{\text{SC}}$ is defined similar to $\rightarrow_{\text{TSO}}$, but the rule (STORE) is replaced by the rule (SCSTORE).

$$\text{(SCSTORE)}\ \frac{\texttt{<inst>} = \text{mem}[r] \leftarrow r', a = val(r), v = val(r')}{(pc, val, buf) \rightarrow_{\text{SC}} (pc', val[a := v], buf)}$$

Note that the buffer will never be used, i.e. early reads and updates from the buffer never occur.

a) Explain the following statement and argue that it is true: There is a correspondence between all executions of a multi-threaded program under SC and the single execution of all single-threaded programs obtained by shuffling the source code of the threads.

b) Let *prog* be a program. We define fency(*prog*) as the program that we obtain from *prog* by inserting an mfence instruction directly after every store operation (i.e. mem[$r$] $\leftarrow r'$).

   Argue whether the following statement is correct: The program *prog* executed under SC has the same behavior as fency(*prog*) does under TSO.

   Here, you may use control-state reachability (see below) as a suitable definition for "having the same behavior".

**Exercise 2: SC reachability**

The (control-state) reachability problem for SC is defined as follows.

**SC-Reachability**

**Given:** Program *prog* over DOM, program counter *pc*

**Decide:** Is there a computation $cf_0 \rightarrow^*_{\text{SC}} (pc, buf, val)$ for some *buf*, *val*?

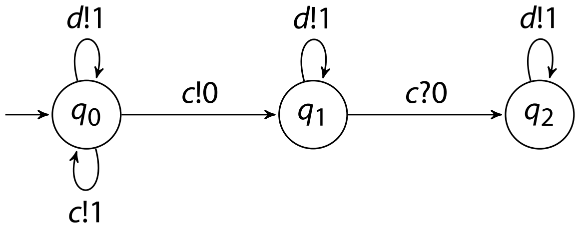
a) Reduce SC-Reachability to Petri net coverability. Explain which places are needed by the net, and how each instruction in the program can be simulated by Petri net transitions.

b) Conclude that SC-Reachability can be solved in PSPACE. Here, you may assume that the size of DOM is encoded in unary.

**We wish all of you a Merry Christmas ...**

**Exercise 3: Expand, Enlarge and Check**

Consider the following lossy channel system *LCS*:




together with $\Gamma = \{(q_0, \varepsilon), (q_1, \varepsilon), (q_2, \varepsilon)\}$ and limit domains

$$L_0 = \left\{\top, \left(q_0, \begin{pmatrix} 1^* \\ \varepsilon \end{pmatrix}\right), \left(q_0, \begin{pmatrix} \varepsilon \\ 1^* \end{pmatrix}\right), \left(q_1, \begin{pmatrix} (0+1)^* \\ 0^*.1^* \end{pmatrix}\right), \left(q_1, \begin{pmatrix} (0+1)^* \\ 1^*.0^* \end{pmatrix}\right)\right\}$$

$$L_1 = L_0 \cup \left\{\left(q_0, \begin{pmatrix} 1^* \\ 1^* \end{pmatrix}\right), \left(q_1, \begin{pmatrix} 1^*.(0+\varepsilon) \\ 1^* \end{pmatrix}\right), \left(q_2, \begin{pmatrix} \varepsilon \\ 1^* \end{pmatrix}\right)\right\}.$$

a) Compute *Over*$(LCS, \Gamma, L_0)$. Provide an execution tree.

b) Compute *Over*$(LCS, \Gamma, L_1)$. Argue why configuration $\left(q_2, \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix}\right)$ is not coverable.

**Exercise 4: Ideals**

Let $(C, \leqslant)$ be a wqo. An **ideal** (with respect to $\leqslant$) is a set $\mathcal{I} \subseteq C$ that is non-empty, downward closed and directed. Directed means that for any $x, y \in \mathcal{I}$, there is $z \in \mathcal{I}$ such that $x \leqslant z, y \leqslant z$.

a) Let $(A, \leqslant_A)$, $(B, \leqslant_B)$ be wqos and let $(A \times B, \leqslant_\times)$ be the product wqo. Show that a set $\mathcal{J} \subseteq A \times B$ is an ideal (wrt. $A \times B$) if and only if it is of the shape $\mathcal{J} = \mathcal{I}_A \times \mathcal{I}_B$ where $\mathcal{I}_A \subseteq A$ and $\mathcal{I}_B \subseteq B$ are ideals (wrt. $\leqslant_A$ resp. $\leqslant_B$).

   *Hint:* For one direction, prove that $\mathcal{J} = \mathrm{proj}_A(\mathcal{J}) \times \mathrm{proj}_B(\mathcal{J})$, where proj denotes the projection (e.g. $\mathrm{proj}_A(a, b) = a$).

b) Show that the ideals of $(\mathbb{N}, \leqslant)$ are $\mathbb{N}$ itself and the sets of the shape $n\downarrow$ for $n \in \mathbb{N}$. Use a) to conclude that the ideals of $(\mathbb{N}^d, \leqslant_d)$ are exactly the sets of the shape $M_\omega \downarrow$, where $M_\omega \in \mathbb{N}^d_\omega = (\mathbb{N} \cup \{\omega\})^d$ is a generalized marking (as they occur in the coverability graph).

c) Prove that the set of ideals is always an adequate domain of limits. You may use the following fact without proof: Any downward-closed set $D \subseteq C$ has a finite ideal decomposition, i.e. a finite set of ideals $\mathcal{I}_0, \ldots, \mathcal{I}_k$ such that $D = \bigcup_i \mathcal{I}_i$.

   *Remark:* In fact, it is also effective in many cases. For example, for LCS resp. the Higman's subword ordering, the set of products (as in the definition of *sres*) is the set of ideals and also an effective adequate domain of limits.

**... and a happy New Year!**