Concurrency theory
Exercise sheet 3
TU Braunschweig
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Out: November 8 Due: November 14

Submit your solutions until Wednesday, November-14, 12:00 am. You may submit in groups up to three persons.

Exercise 1: SRE Inclusion

Use the algorithm given in the lecture to check whether the following SRE inclusions hold:

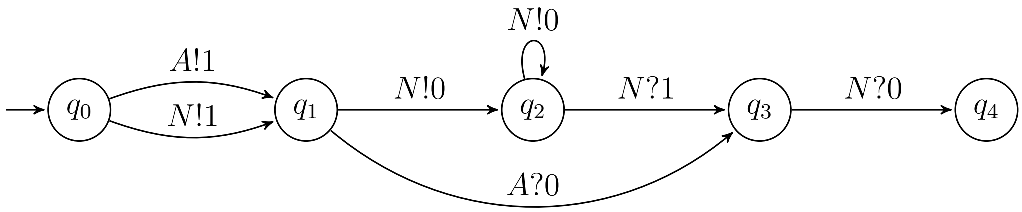
(a) $(a+n+s)^*(t+a+n)^* \subseteq (s+a+n+t+a)^*$

(b) $(r+\epsilon)(p+\epsilon)(n+t)^* \subseteq p^*(r+\epsilon)(s+\epsilon)(n+t)^* + (p+\epsilon)r^*(n+e+t)^*$

(b) $(r+\epsilon)(p+\epsilon)(n+t)^* \subseteq (p+r+e)^*(s+\epsilon)(n+t)^*$

Exercise 2: Coverability of lossy channels

Consider the lcs depicted in the figure below.



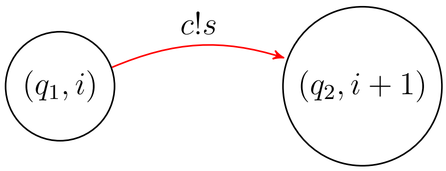
Determine if configurations $\left(q_4, \begin{bmatrix} 0 \\ \varepsilon \end{bmatrix}\right)$ and $\left(q_4, \begin{bmatrix} \varepsilon \\ 1 \end{bmatrix}\right)$ are coverable using the known procedure. (In the vectors, the upper entry is the N entry and the lower entry is the A entry.)

Exercise 3: Generalised Lossy Channel Systems

Consider the following variation of a lcs: assume one of the symbols $s \in M$ can not be lost during send/receive by any channel but that a channel can contain at most $k \in \mathbb{N}$ symbols s.

A transition that wants to send the $k+1$st symbol s is blocked. Such a generalized lcs can be represented by a standard lcs using as states the Cartesian product $Q \times \{0, \ldots, k\}$ where Q is the set of states of the original system.

The resulting lcs transitions are schematically represented below (for $0 \leqslant i < k$).



You are asked to give an implementation of $(q_1, i) \xrightarrow{c!s} (q_2, i+1)$ by several lossy transitions. Your model should check that precisely i symbols s are present in the channel c before appending the extra s.

[Hint: Take $M \cup \#$ as the alphabet of the resulting lcs]

Exercise 4: Lossychannel with Natural numbers

Consider another type of lcs $L = (Q, q_0, \{c\}, M, \rightarrow)$ with c a channel carrying natural numbers as content, i.e., $M = \mathbb{N}$. Take the ordering $\leqslant^* \subseteq M^* \times M^*$ given in Higman's lemma.

(a) Prove that $(Q \times M^*, \lhd)$, with $\lhd$ defined by $(q, w) \lhd (q, w')$ iff $w \leqslant^* w'$, is a wqo.

(b) The transitions in L are given by $q \xrightarrow{!n} q'$ and $q \xrightarrow{?n} q'$ with $n \in \mathbb{N}$. The first appends n to the channel, the second receives a number $n' \geqslant n$ with $n' \in \mathbb{N}$ from the head of the channel. The channel is supposed to be lossy. Formalise the transition relation between configurations.

(c) Prove that $((Q \times M^*, (q_0, \epsilon), \rightarrow), \lhd)$ is a wsts.