

# Concurrency theory

## Exercise sheet 7

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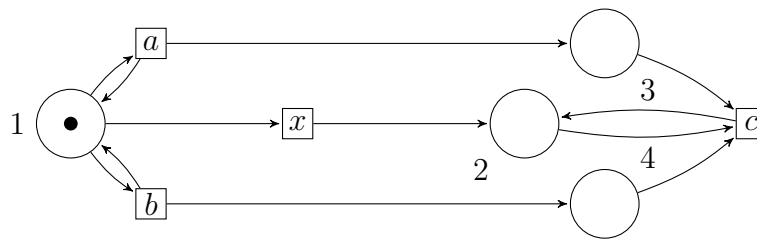
Out: December 12

Due: December 19

Submit your solutions until Wednesday, December 19, 12:00 am. You may submit in groups up to three persons.

### Exercise 1: Rackoff's bound

Consider the Petri net  $N = (\{1, 2, 3, 4\}, \{a, b, c, x\}, \mathbf{i}, \mathbf{o})$  with multiplicities as depicted below. The initial marking of interest is  $M_0 = (1, 0, 0, 0)^T$  and the final marking is  $M_f = (1, 0, 10, 100)^T$ .



Compute the values  $m(3, M_0)$  and  $f(3)$  and argue why they are correct.

### Exercise 2: Counter programs

You may use additional counter variables to solve these problems. In each part of this exercise, you may use the previous parts as subroutines.

Let  $n$  be some fixed number.

- Present a counter program  $\text{Set}_n(x_j)$  that sets the value of counter variable  $x_j$  to  $n$ .
- Present a counter program  $\text{Double}(x_j)$  that doubles the current value of counter variable  $x_j$ .
- Present a counter program  $\text{Power}_n(x_j)$  that sets the value of counter variable  $x_j$  to  $2^n$ .
- Present a counter program  $\text{Square}(x_j)$  that squares the value of counter variable  $x_j$ , i.e. the new value is  $v^2$ , where  $v$  is the old value.

In each part of this exercise, argue briefly that your program is correct.

### Exercise 3: Communication-free Petri nets and SAT

A **communication-free Petri net** (or **BPP net**) is a Petri net in which each transition consumes at most one token, i.e. we have  $\forall t \in T: \sum_{p \in P} i(t, p) \in \{0, 1\}$ .

Show that the coverability problem for communication-free Petri nets is NP-hard by reducing SAT. To this end, show how to construct in polynomial time from a given Boolean formula  $\varphi$  in conjunctive normal form communication-free Petri net  $(N, M_0, M_f)$  such that  $M_f$  is coverable if and only if  $\varphi$  is satisfiable.