Concurrency theory
Exercise sheet 1

TU Braunschweig

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Out: October 30 Due: November 5

Submit your solutions until Tuesday, November 5, during the lecture. You may submit in groups up to three persons.

Exercise 1: The Ackermann function

a) The three-argument Ackermann function φ is defined recursively as follows.

$$\begin{aligned}
&\varphi\colon \mathbb{N}^3 \to \mathbb{N} \\
\varphi(m,n,0) &= m+n \\
\varphi(m,0,1) &= 0 \\
\varphi(m,0,2) &= 1 \\
\varphi(m,0,x) &= m && \text{for } x > 2 \\
\varphi(m,n,x) &= \varphi(m,\varphi(m,n-1,x),x-1) && \text{for } n > 0 \text{ and } x > 0
\end{aligned}$$

Formally prove the following equalities (e.g. using induction):

$$\varphi(m,n,0) = m+n, \qquad \varphi(m,n,1) = m \cdot n, \qquad \varphi(m,n,2) = m^n.$$

b) Nowadays, one usually considers the following two-parameter variant.

$$\begin{aligned}
&A\colon \mathbb{N}^2 \to \mathbb{N} \\
A(0,n) &= n+1 \\
A(m,0) &= A(m-1,1) && \text{for } m > 0 \\
A(m,n) &= A(m-1,A(m,n-1)) && \text{for } m > 0 \text{ and } n > 0
\end{aligned}$$

For example, we have

$$A(1,2) = A(0,A(1,1)) = A(0,A(0,A(1,0))) = A(0,A(0,A(0,1))) = A(0,A(0,2)) = A(0,3) = 4\,.$$

Similar to this computation, write down a full evaluation of $A(2,3)$.

Exercise 2: Konigs Lemma

A finitely branching tree $\mathcal{T}$ is simply defined as a connected directed graph which satisfies the following properties.

- Every vertex in the tree is either a root node, internal node or a leaf node.
- There is exactly one root node, which has no incoming edges and has finitely many outgoing edges.
- Every internal node has exactly one incoming edge and finitely many outgoing edges.
- Every leaf node has exactly one incoming edge and no outgoing edges.

Prove that every finitely branching infinite tree has an infinite path. A path is a sequence of vertices in which the adjacent vertices are related by an edge.

Exercise 3: Petri net constructions

a) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M'_0, M'_f) with $M'_0(p) = 0$ for all places but a single place p' with $M'_0(p') = 1$ and $M'_f(p) = 0$ for all places such that $M_f \in R(N, M_0)$ iff $M'_f \in R(N', M'_0)$. Recall that $R(N, M_0)$ denotes the set of all reachable markings of N, starting from M_0.

b) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M'_0, M'_f) such that M_f is coverable from M_0 in N iff M'_f is reachable from M'_0 in N'.

c) Construct a Petri net N with only 3 places, a marking M_0 and markings $M_{c \wedge r}, M_{\neg c \wedge \neg r}$ and $M_{c \wedge \neg r}$ such that

- $M_{c \wedge r}$ is reachable and coverable from M_0,
- $M_{\neg c \wedge \neg r}$ is neither reachable nor coverable, and
- $M_{c \wedge \neg r}$ is coverable, but not reachable.

In each part of this exercise, argue briefly that your construction is correct.