Out: April 5   Due: April 10

Submit your solutions until Monday, April 10, 14:00, in the box in the Institute for Theoretic Computer Science, next to office 343. You may submit in groups up to three persons.

Exercise 1

Complete the tree from Example 3.3 from the lecture notes, i.e. draw the full tree of plays for the initial state $(2, 2, 1)$, where we assume that player 1 has to move first. For every node, write down the Nim sum. Furthermore, mark all winning states in the tree.

Exercise 2

Prove Lemma 3.9 from the lecture notes: Let $(c_1, \ldots, c_k)$ be an unbalanced state. There is a successor state (i.e. a state to which we can go with one single move) that is balanced.

Hint: Consider the smallest index j such that $\mathrm{Nim}\Sigma(c_1, \ldots, c_k)_j$ is odd. (Note that "smallest" means that the corresponding bit is most significant.) Prove that there is an index i with $c_{ij} = 1$ that can be modified to get to a balanced state.