SS 2012 30.5.2012

Exercises to the lecture Logics
Sheet 4

Jun.-Prof. Dr. Roland Meyer Due 12.6.2012 12:00 Uhr

Exercise 4.1 [Resolution Calculus]

a) Let K_1, K_2 be clauses and I be a literal with $I \in K_1$ and $\neg I \in K_2$. Show that $\{K_1, K_2\} \models \text{Res}_I(K_1, K_2)$.
b) Prove the correctness of the resolution calculus, i.e. show that for any formula F and any clause K with $F \vdash_{\text{Res}} K$, we have $F \models K$.
c) Using the Resolution Calculus, show that $(((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow \neg(\neg r \wedge p))$ is a tautology.

Exercise 4.2 [Dual Formulae]

a) Calculate $d(\neg((p \wedge q) \vee (r \wedge \neg s)))$ step-by-step according to Definition 2.19.
b) For each valuation φ, let φ' defined by $\varphi'(p) = 1 - \varphi(p)$ for each variable p. Show that for any formula A, we have $\varphi'(d(A)) = 1 - \varphi(A)$.
c) Deduce from b) that for any formula A, the following holds: A is a tautology if and only if $d(A)$ is unsatisfiable.

Exercise 4.3 [Negation Normal Form]

Using structural induction, prove that for any formula there is an equivalent formula in negation normal form. *Hint:* In order to make the induction work, show by induction that A as well as $\neg A$ has a negation normal form.

Exercise 4.4 [Tableaux]

Let Σ be a set of formulas and p, q be atomic formulae with $\Sigma \vdash_\tau p$ and $\Sigma \vdash_\tau p \rightarrow q$. Prove that then $\Sigma \vdash_\tau q$.

Delivery: until 12.6.2012 12:00 Uhr into the box next to room 34/401.4